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Information, Communication and Many-Valued Logic

The exact scientific treatment of the problems of Information and Communication is relatively new. It has made its first impact - as should be obvious – in the mathematically orientated sciences. However, the theory of communication which C. C. Shannon presented in 1948, [1] although satisfactory for its purpose, requires additional work by logicians and semanticists to permit its full application in the Humanities as well as in Philosophy. Shannon's theory excludes the concept of Meaning from Information in order to make the phenomenon responsive to the mathematical means which are today available. If we want to introduce meaning we have to assume that two persons, who contact each other over a channel of communication with the explicit purpose to exchange meaning full information, are is possession of a prearranged code to interpret the signals which they receive. We all have such codes at our disposal in our common cultural heritage and in the social institutions (e.g. education) which represent the background of our existence.

The cultural and philosophic problem of Information and Communication is as old as the earliest endeavors of Man to reflect about himself, his associates and the human society which sheltered and constrained him. In modern times the issue has become even more acute. Theoretical as well as practical motives to study the nature of communication and, if possible, to improve on it have attained a new urgency. On the theoretical side it is the rise of philosophic anthropology which poses questions that can only be answered if we acquire a deeper understanding of what Information means in relation to human consciousness and self-awareness. On the practical side it is the present clash of the diverse ideologies and world conceptions which have risen in the East and the West. The technical progress which has drawn together all the different human societies and cultures that have grown up on this planet provokes an interchange of ideas and a communication of as yet mostly irreconcilable view-points. Under the circumstances we cannot evade the task of developing a reliable theory of communication which shows us the means to transmit unequivocally information about the cultural aspects of human life.

As has been pointed out above, the information and communication theory in its present stage is not yet fully capable of doing the job. This is mostly due to its severest limitation: It is not able to define the relation between information and meaning! In fact, the success of the theory which Shannon and his collaborators developed depends on a careful separation of the two and on the exclusion of the concept of meaning from the formulas describing the laws that govern the transmission of information from its

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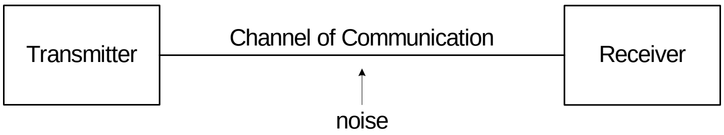
1 Cf. Claude Shannon: "A Mathematical Theory of Communication," Bell Syst. Tech. Journal 27. pp. 379-423 and 623-656.

source to its recipient. It is obvious that this approach is inadequate both for philosophic anthropology and for the theory of culture which the Humanities try to develop.

The question is now what causes the shortcomings of the present mathematical theory of communication? In order to give an answer we shall draw the simplest possible scheme to illustrate what the communication theories talk about.



This drawing has essentially three parts. On the left side is the transmitter which may originate and transmit a message. The center part represents the flow of the communication as well as the medium which carries it. This flow may be disturbed and thus the message distorted by a "noise" due to the specific character of the medium. The more involved patterns which are usually drawn are not necessary to illustrate our argument.

Our argument runs as follows: the present mathematical theory of communication is neither sufficient in Philosophy nor in the Humanities (cultural sciences) because the observer who describes the process and develops his formulas cannot be identified with the logical position of the transmitter and he is also incapable of occupying the logical stand-point of the receiver. His status in the theory places him entirely outside the system which he observes. This has intricate logical consequences if we assume that the author of a communication formula takes the position that transmitter as well as receiver function with the same, or at least approximately the same, degree of logical organization as he himself as an observer from the outside. As long as this assumption is not made the positions of transmitters and receivers are logically reversible.[2] In other words nothing is logically changed in our description of the process if we permit the communication to flow in the opposite direction. The implication is, that there is no formal, logical distinction between transmitter and receiver but only between the process of communication and the systems which send messages to each other. To put it differently: the present logical structure of communication and information theory is strictly two-valued. And this two valuedness is by no means impaired by the fact that a given message may be characterized by different measures of probability. To note this is important since the erroneous opinion still persists that measures of probability may be equated with the number of values attributed to a logic.[3]

However, we shall now assume that transmitter, receiver and observer all represent human persons. This makes them relatively independent and subjectively active centers of self-reflection. In this case the observer may (or he may not) place himself spontaneously outside the system which comprises transmitter, communication channel and receiver. We shall now assume that, for a given case of sending a message, the observer will be identical with that part of the system which acts as receiver. This

2 The asymmetry of equivocation, $H_x(y)$ may be ignored in this context. Only the case of $H(x) + H(y) = H(x, y)$ is considered.

3 Cf. H. Reichenbach, *Experience and Prediction*, Chicago, 1947, p. 326 ff. Also: G. Günther, *Cybernetic Ontology and Transjunctional Operations*, Self-Organizing Systems 1962 (Ed. Yovits, Yacobi, Goldstein) Spartan Books, Washington, 1962.

complicates the situation. On one logical level, which is strictly two-valued we may ignore the additional self-reflective capacity of the receiver and treat him as an "it" which receives messages as any properly designed piece of hardware may. But on a second level of logic all our two-valued data will be inextricably enmeshed in the process of self-awareness which transforms the former "it" into a "he" or "she." If this situation is taken into account the original two-valued reversibility between transmitter and receiver breaks down, because the system which was originally designated as receiver may now observe itself changing its role and becoming the transmitter. What should be understood is that this self-observation is, logically speaking, quite different from the observation of a neutral observer who watches the process from the outside. If we want to express the difference in terms of identity, we may say: the neutral observer who is neither identical with the transmitter nor with the receiver formulates his observations in terms of hetero-reflexive identity. The observer who, at a given time, is either identical with one or the other terminal of the communication channel expresses his finding in terms of *self-reflexive identity*. This difference has been known to transcendental logic for more than one and a half centuries. It has also been known that our two-valued classic logic deals only with concepts which imply hetero-reflexive identity. In fact this is, in the most abbreviated form, the lesson Kant's *Critique of Pure Reason* teaches the student of formal Aristotelian logic.

In order to establish the distinction between hetero-reflexive and self-reflexive identity more clearly let us go back once more to our stipulated situation of the non-neutral observer who is identical with either the transmitter or the receiver. He must be located at either one or the other terminal of the communication channel. It is impossible for him to be located at both ends at the same time. It follows that he watches what happens at both ends in very different manners. At one terminal the events which take place occur within the dimension of his "private" self-reflection. He possesses, so to speak, insight into them. They constitute events within the confines of his "Bewusstseinsraum". But what takes place at the other terminal are occurrences within the outside world. To put it differently: the symmetric exchange relation between transmitter and receiver which the "neutral" observer may establish in a logic of hetero-identical terms is disturbed for the non-neutral observer who understands the events in terms of self-reflection. For the neutral observer, the two terminals of a communication channel are logically equivalent on the basis of a formal exchange relation in two-valued logic. For the observer who has identified himself with one of the two terminal points of the channel of communication these points cease to be logically equivalent. *This situation requires at least a three-valued logic*! Instead of our former dichotomy of communication and participants in the communication process we have now to introduce a trichotomy of a) transmitter, b) communication, c) receiver. And self-reference will be attributed to either a) or c) but not to both. To put it differently: we have two logical choices. We may state the equivalence a) ≡ c). In this case the process of communication is reversible. This is the way the observer from the outside looks at it. Or we may say a) ≢ c). If we do so we assume that the observer is involved in the process of communication and at least partly identical with the receiver. The result of this reflexive identity is that the process of communication becomes irreversible and it is impossible to deal with it in terms of a two-valued logic.

All this has been known to students of transcendental, self-reflective logic for a long time. So it is reasonable to ask: why has many-valued logic never been seriously introduced in the Humanities and in philosophic anthropology where the problem of communication-with-self-reference is paramount.[2] The answer is that any logician who has no illusions will admit that he does not understand what a many-valued logic really is.

For more than 40 years attempts – initiated by Łukasiewicz and Post have been made to introduce many-valued theories of thinking into Philosophy. So far all these attempts have failed as has been testified by C. I. Lewis, I. M. Bochenski, H. A. Schmidt and others. A German logician, von Freytag-Löringhoff, has drawn attention to a practical difficulty. In two-valued logic we have to deal with 16 binary constants for what is usually called the propositional calculus. It is within the realm of possibility to investigate each of these constants individually. A great deal of work has been done in this direction although it is by no mean finished. But if we add just one more value the number of binary constants increases to 19683 and von Freytag-Löringhoff has pointed out that it is impossible to process each of these constants in the same manner as is customary in two-valued logic. The difficulty is even greater than the German logician suspects.

If we advance into the realm of a three-valued logic not only binary constants are to be taken care of. We have also to introduce a third variable and investigate ternary functions. This raises the number of accountable constants to 19683 + 3^{27} which is approximately 10^{14}. But as we have shown elsewhere [4] that is by no means sufficient. A three-valued logic is morphogrammatically incomplete and we are forced to proceed to four values and four variables to accommodate them. In other words: we have to add to the number of binary and ternary four-valued sequences 4^{256} quaternary functions which is approximately 10^{153}. This number is already well beyond the order of the biggest astronomical numbers which hardly exceed 10^{100}. Unfortunately the introduction of a four-valued logic satisfies only the minimum requirement of the morphogrammatic theory of logic. It can be said with an assurance which closely approaches certainty that even the description of the complexity of self-reflective structures which are embodied in the simplest forms of living cells requires such functions K and T. For values we use the first three numbers of our decimal system. We stipulate also that 1 and 2 may represent the values of our traditional, classic logic.

I

p	q	K	T
1	1	1	1
2	1	2	3
3	1	3	2
1	2	2	3
2	2	2	2
3	2	3	1
1	3	3	2
2	3	3	1
3	3	3	3

[4] G. Günther, *Cybernetic Ontology and Transjunctional Operations*, in: Self-Organizing Systems, M. C. Yovits, G. T. Jacobi G. D. Goldstein (eds.), Washington D. C. (Spartan Books) 1962, 313-392

K is easily recognized as the normal conjunction of a three-valued system and T is another one out of the reservoir of 19683. Since it has very specific properties which single it out we have given it a name. We call it "transjunction": The basic distinction between value-sequences of the K- and the T-type is easy to grasp. Wherever the values of the variables p and q differ K takes one of the two values. The function, so to speak, chooses its value from an alternative that is "offered" by p and q. The function T follows exactly the opposite law. Whenever the states of p and q differ T never takes one of the values which are provided by the variables. In a manner of speaking T refuses the offer and "rejects" the alternative of values which characterize 6 of the states of the function. In the case of a three-valued system only one "rejection-value" will be available for a given state of p and q. In the case of the alternative $1 \longleftrightarrow 2$ it is 3. If p and q differ by $2 \longleftrightarrow 3$ it must be 1, and if the alternative is established by $1 \longleftrightarrow 3$ then 2 will be the value which rejects the state of p as well as of q.

It is obvious that the distinction between value-sequences where the function always "accepts" one, of the values which p and q competitively offer the value-sequences where their offers are either totally or partially rejected permits us to define a dichotomy of the 19683 constants. This is not very much, however, and we have to go considerably farther. We shall have at least a seven-valued logic. To describe the structural complexity of a high culture like Western Civilization we would have to introduce a number of logical constants of self-reflection which is fantastically beyond the competence of a seven-valued system. The number of values and variables required for an adequate description of man's present historical status would probably have to be in the neighborhood of that number of neurons of the brain which has been engaged in the task of bringing mankind up to its present level of History.

It is obvious that all present logical methods used to analyze the self-reflective structure of History in general and of individual civilizations as means of communication for human beings are of an almost unbelievable inefficiency. They are based on 16 logical constants of a two-valued, morphogrammatically incomplete, propositional calculus. But it is utterly impossible to handle even the smallest many-valued systems with the traditional methods of logical analysis, as von Freytag-Löringhoff has pointed out. The German scholar implies that no other methods are available.

This is, however, an unwarranted assumption and we shall devote the rest of this essay to point out the direction in which we have to look for a solution of the problem. At the moment, logicians are even afraid of the number 19683 in connection with binary constants in the propositional calculus. But astronomers are not worried by numbers in the neighborhood of 10^{100}. And why should they! They use a system of handling numbers which is so simple that children in school learn it long before they enter a university. It is a place-value system with a predetermined range in which every number is iterated alter it has run the range. The extent of this range in our conventional way of counting is provided by the decimal place-value system of numbers. We have shown elsewhere[5] that this idea is, with simple modifications, applicable to many-valued systems. It will be easy to demonstrate that the interpretation of many-valued systems as place-value arrangements of our two-valued classic logic provides us with a clue how

5 G. Günther, *Die Logik des Seins und die Logik der Reflexion*, Zeitschrift für philosophische Forschung XII, 3 (1958), pp. 360-407.

to classify and thus to provide identification for the 19683 binary constants of a three-valued system.

A three-valued binary function has nine different states for any function that may hold between its two variable p and q. In the following Table_I we have written down our functions K and T now dissected according to the place-value theory into 3 two-valued subsystems:

II

p	q	K_1	K_2	K_3	T_1	T_2	T_3
1	1	1		1	1		1
2	1	2			3		
3	1			3			2
1	2	2			3		
2	2	2	2		2	2	
3	2		3			1	
1	3			3			2
2	3		3			1	
3	3		3	3		2	3

It is easy to see that the morphogrammatic structure of K as a four-place value sequence (as it is used in the propositional calculus of classic logic) turns up in 3 different places. The different value occupancy as embodied by 1222, 2333 and 1333 serves only to distinguish the places logically from each other. But what really counts is that the structure which is generated by the distribution of values over four places is repeated three times.

One feature stands out: the different role the values play in the positions 1, 5 and 9 of the sequence as contrasted with the values in the positions 2, 3, 4, 6, 7 and 8. If one of the latter values is transformed into a lower or higher value only the morphogrammatic structure of the four-place sequence which houses it is affected. But a change in the value-occupancy in the positions 1, 5 and 9 always concerns two morphograms. An alteration in the first position of K engages K_1 and K_3. An alteration in the fifth position of K is relevant for K_1 and K_2. Any change in the last position is the affair of K_2 as well as of K_3. We shall now say that the values in positions 1, 5 and 9 of Table_II represent the "frame" of the sequence and those in 2, 3, 4, 6, 7 and 8 the "core" of the sequence. Generally speaking, any m-valued sequence has m frame-positions. The rest belongs to the core. The negation of an individual value occupying a frame-position may under certain circumstances alter the structure of the frame. A negation of an entire value sequence (frame plus core) will never do so.

We are now ready to demonstrate the principles of classification of all binary logical constants of any m-valued logic. We shall start with the rather trivial case of the two-valued system. In order to do so we write down all 16 value-sequences in a specific order which follows the classification principles we intend to employ. It stands to reason that the concept of the rejection-value will play no part in the case of classic logic since only two values are available. Table_III displays the order of classification, which, incidentally, involves another dichotomy between what we shall call "irreflexive" and "reflexive" value sequences. This dichotomy corresponds to the distinction of position and negation in classic logic. In the simple case of two-valued systems the dichotomy presents no problem. If p and q both have the value 1 we call the ensuing sequence, no matter what the value-occupancy of the subsequent positions, irreflexive or positive. In all the other cases it is reflexive or negative. In many-valued cases, however, it is not so simple to obtain the proper dichotomy. Table_III will help to elucidate the procedure.

III

a	b	c	d	e	f	g	h	
1	1	1	1	1	1	1	1	
1	2	1	2	1	2	1	2	irref.
1	1	2	2	1	1	2	2	
1	1	1	1	2	2	2	2	
2	2	2	2	2	2	2	2	
2	1	2	1	2	1	2	1	refl.
2	2	1	1	2	2	1	1	
2	2	2	2	1	1	1	1	

The dotted oblongs which enclose the second and third positions of the columns separate the core of the sequences from their frames. We notice at once that the value-sequences of the propositional calculus of classic logic require two frames. In the upper group the value-occupancy of the frames (first and fourth position) is irreflexive. In the lower group it is reflexive. Moreover, we discover – and this is of great significance – that in order to obtain all ref. value sequences it is sufficient to negate the frames of the irref section of Table_III. This involves a subtlety. If we negate the frame of a-irref we do, of course, not obtain a-ref but d-ref. And the negation of the frame of b-irref produces c-ref. These data play a subtle role in the relations between morphograms and their potential value-occupancies. Here they may be ignored because our discussion refers only to value-systems. It is sufficient to acknowledge that in order to obtain all negated value sequences of two-valued logic we have not to negate all values but the value-occupancies of the frames. This will be equally the case for any m-valued system. We shall now demonstrate the situation for three-valued binary sequences.

From Table_II we are already aware that these sequences have frames with three value positions. The number of possible frames is five as Table_IV displays:

IV

$irref^1$	$Irref^2$	$irref^3$	$irref^4$	$irref^5$
1	1	1	1	1
:	:	:	:	:
:	:	:	:	:
1	1	2	2	2
:	:	:	:	:
:	:	:	:	:
1	2	1	2	3

We stipulate that the value occupancies of the frames in Table_IV represent their irreflexive character. Any other value-occupancy obtained by negating the values turns them into reflexive frame-structures. It is a significant departure from traditional logic that we now have to distinguish five degrees of irreflexibility for the frame of a value-sequence. The choice of the values for irreflexivity is by no means arbitrary. We stipulate that for the n-th position of the frame the occupying value can never be higher than n if irreflexivity is claimed.

To each irref-frame belongs a number of reflexive versions. In the case of $irref^1$ there are only two but in all the other cases of Table_IV the number is five. It follows that a three-valued logic, having five frames is capable of producing 27 value-occupancies for them. On the other hand, the number of possible value-occupancies for the six core-positions amounts to $3^6 = 729$ and $27 \times 729 = 19683$ which is, as we remember, the number of all binary constants of a three-valued "propositional calculus."

We are now in a position to classify all binary sequences of three-valued logic by distinguishing them according to the following characteristics:

value	:	acceptance
		rejection
place	:	$1 \longleftrightarrow 2$
		$2 \longleftrightarrow 3$
		$1 \longleftrightarrow 3$
frame	:	$irref^{\mathbf{1} \dots \mathbf{5}}$
		refl. (N_1 and N_2)

N_1 and N_2 indicate the two negational operators of a three-valued system

p	N_1 p	N_2 p
1	2	1
2	1	3
3	3	2

which may be combined in a suitable manner [6] to obtain all possible permutations of the value-sequence 1, 2, 3.

However, some of the classes of value-sequences obtained by using the principles enumerated above are still fairly large and unwieldy. We, therefore, introduce one more subdivision for acceptance - as well as rejection values. It is the common property of all states of the variables which determine the value-occupancy of the core of a function that at least one of the variables p, q, r... must display a value which differs from the values which the other variables show in this particular case. If only two variables are available for the accommodation of three values we may distinguish between what may

6 G. Günther, *Cybernetic Ontology and Transjunctional Operations*, in: Self-Organizing Systems, M. C. Yovits, G. T. Jacobi G. D. Goldstein (eds.), Washington D. C. (Spartan Books) 1962, 313-392

be called symmetrical and asymmetrical acceptance and rejection. The following Table_V

V

p	q	sym		asym		sym	asym			
		acceptance				rejection				
1	2	1	2	1	2	3	1	3	2	3
2	1	1	2	2	1	3	3	1	3	2

shows for the case p = 1 and q = 2 and vice versa the possible value choices for symmetrical and a-symmetrical acceptance and rejection. If the acceptance is symmetrical both states of p, q are accepted by the same value. If it is a-symmetric case the value character of the acceptance differs. The same goes for rejection. Here an even finer distinction could be made according to which of the two values 1 or 2 makes the rejection asymmetrical. In our present discussion we shall ignore this possibility which is of considerable usefulness in more comprehensive systems of logic. We only intend to show the general principles of classification of the "propositional" constants of m-valued orders without exploiting all possibilities of subdivision. What will be ignored are additional principles of classification which arise from a further analysis of the core of a value-sequence. The Table_VI of classification then looks as follows:

VI

Classification of the 19683 Binary Constants of Three-valued logic.

A: Constants without rejection-values:

I. Symmetrical choice of values:

Frame :

Irref. 1	8	constants
ref. 1	2 × 8 = 16	constants
Irref. 2	8	constants
ref. 2	5 × 8 = 40	constants
Irref. 3	8	constants
ref. 3	5 × 8 = 40	constants
Irref. 4	8	constants
ref. 4	5 × 8 = 40	constants
Irref. 5	8	constants
ref. 5	5 × 8 = 40	constants

(In order to shorten the following parts of the Table we shall from now on lump together frames 2 to 5 since they have always the same number of reflexive value-occupancies.)

II. A-symmetrical choice of values:

Frame :

Irref. 1	56	constants
ref. 1	2 × 56 = 112	constants
Irref. 2-5	4 × 56 = 224	constants
ref. 2-5	4 × 280 = 1120	constants
All constants of A I and A II	1728	constants

B: Constants with rejection values.

I. Symmetrical choice of values:

a) Subsystem 1 $\longleftrightarrow$ 2

Frame :

Irref. 1	4	constants
ref. 1	2 × 4 = 8	constants
Irref. 2-5	16	constants
ref. 2-5	5 × 16 = 80	constants
	108	constants

b) Subsystem 2 $\longleftrightarrow$ 3

dto 108 constants

c) Subsystem 1 $\longleftrightarrow$ 3

dto 108 constants

d) Subsystems 1 $\longleftrightarrow$ 2 and 2 $\longleftrightarrow$ 3

Frame :

Irref. 1	2	constants
ref. 1	2 × 2 = 4	constants
Irref. 2-5	8	constants
ref. 2=5	5 × 8 = 40	constants
	54	constants

e) Subsystems 2 $\longleftrightarrow$ 3 and 1 $\longleftrightarrow$ 3

dto 54 constants

f) Subsystems 1 $\longleftrightarrow$ 2 and 1 $\longleftrightarrow$ 3

dto 54 constants

g) All subsystems

Frame:

Irref. 1	1	constants
ref. 1	2 × 1 = 2	constants
Irref. 2-5	4	constants
ref. 2-5	5 × 4 = 20	constants
	27	constants

All constants of B I : 513 constants

II. Asymmetrical choice of values:

a) Subsystem 1 $\longleftrightarrow$ 2

Frame :

Irref. 1	76	constants
ref. 1	2 × 76 = 152	constants
Irref. 2-5	4 × 76 = 304	constants
ref. 2-5	4 × 380 = 1520	constants
	2052	constants

b) Subsystem 2 $\longleftrightarrow$ 3

dto 2052 constants

c) Subsystem 1 $\longleftrightarrow$ 3

dto 2052 constants

d) Subsystems 1 $\longleftrightarrow$ 2 and 2 $\longleftrightarrow$ 3

Frame :

Irref. 1	98	constants
ref. 1	2 × 98 = 196	constants
Irref. 2-5	4 × 98 = 392	constants
ref. 2-5	4 × 490 = 1960	constants

e) Subsystems 2 $\longleftrightarrow$ 3 and 1 $\longleftrightarrow$ 3

dto 2646 constants

f) Subsystems 1 $\longleftrightarrow$ 2 and 1 $\longleftrightarrow$ 3

dto 2646 constants

g) All subsystems

Frame :

Irref. 1	124	constants
ref. 1	2 ×x 124 = 248	constants
Irref. 2-5	4 × 124 = 496	constants
ref. 2-5	4 × 620 = 2480	constants
	3348	constants
	3349	
All constants of B II :	17442	constants

This classification Table is capable of generalization if either more variables or more values or more of both are introduced. It is asserted that it implies all basic logical concepts which refer to the process of communication. Moreover, it indicates that among the

1728	constants of A I and II
513	constants of B I
17442	constants of B II
19683	constants

there is one, B, I, g, irref 1, which is the key-sequence of the whole system, having very unique properties. (B, I, g irref. 1 is function T of Table_II.)

The fantastic wealth of "propositional" constants in many-valued systems has so far been a serious hindrance in applying these systems to the problem of communication. The classification method presented above should make this task much easier. The process of communication described only in terms of information refers logically exclusively to A-group of our classification. The concept of meaning is connected with the B-group. The union of Information and meaning, however, requires A and B.

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