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Heterarchy – Hierarchy

Two complementary categories of description

1

Info

Although the notion of 'heterarchy' was introduced into science by Warren S. McCulloch[0] nearly 60 years ago, the meaning of this term, as it is given by the definition in ref.[1], and its epistemological consequences have not yet been reflected by the mainstream of the scientific community.

A little experiment using a well-known search engine (e.g., Google) demonstrates that the ratio between the number of hits for 'heterarchy' and its complementary term 'hierarchy' is 1:1,300. In other words, hierarchical structures are definitely preferred by our way of thinking.[2] Looking for a definition of 'heterarchy' leads to an etymological definition that refers to its Greek origin, namely *heteros* (the other, the alien, ...) and *archein* (to reign, to govern, ...), i.e., *under the governance of an alien*.[3] This definition, however, raises the question: "Who is the governor in a hierarchy?", or in other words, how can we understand the complementarity of the two terms?

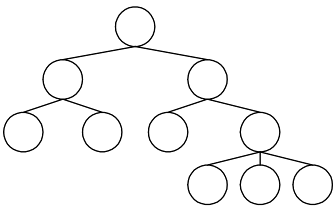

figure_1a

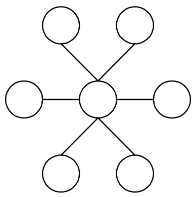

figure_1b

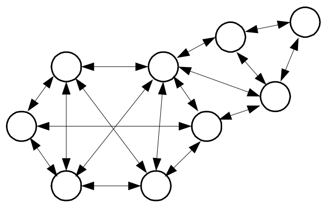

figure_1c

If one considers 'hierarchy' in the context of organizational forms with sub- or super-ordination within different social domains, such as in the armed forces, bureaucracy, politics, religious organizations, etc.[4] then the complementary term 'heterarchy' can be

0 McCulloch, Warren Sturgis - (1898-1969). McCulloch received his M.D. from Columbia University's College of Surgeons and Physicians. He is perhaps best known for "A Logical Calculus Immanent in Nervous Activity", which he co-authored with Walter Pitts. This paper is widely credited as being a seminal contribution to neural network theory, the theory of automata, the theory of computation, and cybernetics. More under: http://www.amphilsoc.org

1 Warren S. McCulloch, A Heterarchy of Values Determined by the Topology of Nervous Nets, Bull. Math. Biophys. 7 (1945) 89-93.
Reprinted in: W.S.McCulloch, Embodiments of Mind, The MIT Press, 1988.

2 One really should not be surprised by the naive universe of the present US-American administration with its creation of the *Axis of Evil*.

3 See for example: http://www.brainydictionary.com/words/he/heterarchy173194.html

4 See for example: http://pespmc1.vub.ac.be/ASC/HIERARCHY.html
(l) A form of organization resembling a pyramid. Each level is subordinate to the one above it. See heterarchy. (Umpleby) (2) An organization whose components are arranged in levels from a top level down to a bottom level. (Arbib) (3) A partially-ordered structure of entities in which every entity but one is successor to at least one other entity; and every entity except the basic entities is a predecessor to at least one other entity. (Rogers) (4) Narrowly, a group arranged in order of rank or class; we

defined as co-ordination. It is this idea from which all definitions of 'heterarchy' result, which refers to a network, for example, '*a form of organization resembling a network or fishnet*' or '*an authority determined by knowledge and function*'.[5] From a scientific point of view neither of these definitions are very precise because one could immediately ask: what about the organizational networks of the organized crime, or what about authorities in hierachically organized institutions like, for example, the catholic church? – Are these authorities without knowledge and function, simply because they are members of an hierarchically structured organization?

It is amazing that both of these somewhat nebulous definitions (*the fishnet* and *the knowledge/function* versions) are taken from the *Web Dictionary of Cybernetics and Systems*. In a dictionary of cybernetics and systems one would expect that at least a short hint be given as to the original publication where this term was introduced. But there is neither a link nor a reference nor any comment dealing with the origin of this term. In order to understand what 'heterarchy' really means scientifically one has to go back to one of the fathers of cybernetics, namely to Warren S. McCulloch and his paper *A Heterarchy of Values...* from 1945.[A1] contains a somewhat longer passage from this paper – a passage which will be analyzed in the following discussion.

There are at least two points in McCulloch's paper which are of far reaching scientific consequence for any formal theory of neural networks, and if considered more generally, for any theoretical formal logical treatment of heterarchical structures. These points are:

1) McCulloch's introduction of a diallel[6] which refers to a logical circle (*circulus vitiosus*) or a logical contradiction, and
2) what he calls a 'value anomaly', which in McCulloch's paper stands for non-transitive (neural) processes.

Before we start to discuss these two points, it should be mentioned that McCulloch's 1945 paper was interpreted logically by Gotthard Günther[7]in the seventies and later in the eighties by Rudolf Kaehr and others[8]. In other words, the following discussion has to be considered a kind of knowledge-recycling task and not the presentation of "brand new

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interpret it to denote a rank arrangement in which the nature of function at each higher level becomes more broadly embracing

5 See for example: http://pespmc1.vub.ac.be/ASC/HETERARCHY.html

6 from Greek: 'diallelos tropos': "the nature of circular moving" in the sense of a *circulus vitiosus*; or "an explication which already includes that what has to be explicated"; or "arguing in a circle".

7 See for example the English written paper of Günther:
Gotthard Günther, *Cognition and Volition. A Contribution to a Theory of Subjectivity*, shortened version in: Cybernetics Technique in Brain Research and the Educational Process, 1971 Fall Conference of American Society for Cybernetics, Washington D.C., 119-135. – printed in: *Beiträge zu einer operationsfähigen Dialektik*, Bd. 2, Felix Meiner Verlag, Hamburg, 1979, p.203-240.
German Translation: *Erkennen und Wollen*, in: G. Günther, *Das Bewußtsein der Maschinen*, AGIS Verlag, ^{3}2002.
Both the English and the German version of *Cognition and Volition* are available on the web: www.vordenker.de

8 See for example:
(a) R.Kaehr & E. von Goldammer, *Again Computers and the Brain*, Journal of Molecular Electronics Vol. 4 S31-S37 (1988)
(b) R.Kaehr & E. von Goldammer, *Poly-contextural Modeling of Heterarchies in Brain Functions*, in: Models of Brain Functions, (R.M.J. Cotterill, ed.), Cambridge University Press, 1989, p. 483-497.
Both papers are available on the web: www.vordenker.de

discoveries", especially if the publications of the so-called *pcl*-group in the last two decades are considered.[[9]]

Because of the fact that the term 'heterarchy' was introduced into science more than 50 years ago and still has not been understood intellectually either by the mainstream of the scientific community or within the circles of cybernetics (particularly within so-called second order cybernetics) one might suspect McCulloch's inheritance of being a puzzling but marginal matter. But McCulloch presented his ideas very clearly, and more importantly, his thinking was of "philosophic profundity" as Günther tells us in *Number and Logos*.[[10]] McCulloch was not only familiar with the philosophy of antiquity but also with very specialized aspects such as "the different views of dialectics entertained in Kant's Critique of Pure Reason and in Hegel's Logic". But this is still not the whole story. It is quite well known that McCulloch was very much interested in Charles S. Peirce's experiments with developing a triadic logic.[[11]] Peirce also was familiar with the philosophy of Kant and Hegel's idea of dialectics. It is a twist of history that the majority of the contemporary brain researchers as well as their colleagues in the cybernetics and Artificial Intelligence camp – at least in the Western part of the world – don't even know the meaning of dialectics. In other words, modern brain and artificial intelligence research has mutated towards a kind of new vulgar materialism.[[12]]

In the following we have to analyze first the meaning of the term 'diallel' and secondly the meaning and the consequences of McCulloch's 'value anomaly' which has to be interpreted as 'non-transitivity' in the context of (neural) processes (with strong emphasis on 'non-transitivity' and on 'process').

Why did McCulloch choose the term 'diallel' instead of 'contradiction', or 'antinomy', or 'paradox', or 'circulus vitiosus'?

This question can be easily answered. The term 'diallel' originates from the Greek *diallelos topos*; it describes a kind of circularity in the context of a process, i.e., the term 'diallel' has a dynamical character (cf. footnote [[6]]). It can be assumed that McCulloch knew the original meaning of the term 'diallel' very well and therefore chose this term to point to the fact that what is being considered are antinomical p r o c e s s e s and not states[[13]]. At this point the question can be raised whether antinomical s t a t e s exist or not. We can assert that paradoxes (or antinomies or a *circulus vitiosus*) cannot be measured in a physical sense. Paradoxes only occur within our interpretations of particular situations; they are never part of the description of physical systems nor of physical states.[[14]] The following examples will demonstrate what we mean by saying "paradoxes only occur within our interpretations of a special situation".

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9 Many of these publications are on the web: PCL-thinkartlab and www.vordenker.de

10 Gotthard Günther, *Number and Logos - Unforgettable Hours with Warren St. McCulloch*, published in vordenker.de since Mar 27, 1997 in: www.vordenker.de

11 McCullochs work has to be judged in front of this background. McCulloch himself was involved in the design of a (graphical) triadic logic (cf.: C.R. Longyear, J. of Cybernetics 2 (1972) 50ff.).

12 We will demonstrate that heterarchically structured processes can never be observed in physics or chemistry.

13 'states' in the sense of a static situation – see also [B1] and [B2].

14 This statement holds for all phenomena that result from the wave-particle dualism and/or similar paradoxes such as the Einstein-Podolsky-Rosen-(EPR-)-phenomenon. All these paradoxes are the result of

- example_1-1:
  Figure_2 shows the end of a two-wired cable. If one is faced with the task of poling the cable correctly one has to mark one of the two wires on both ends of the cable. This is shown in figure_2 for one end of the cable. Now, for a user, i.e., the electrician, both wires are "marked". It would be extremely stupid to attach a small red marker on both wires of the cable in order to distinguish them.

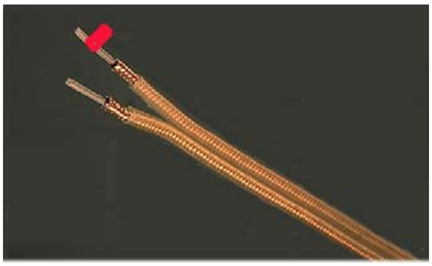

figure_2 : see text

- example_1-2:
  In the following example a word processing program will immediately discover the two syntactical errors but not the third, the semantical error, viz.

This sentense contains three erors.

Obviously our brain is able to discover the third error in the same way as it is able to distinguish the non-marked marked wire from the (non)marked wire, while no logical machine, no computer – without being pre-programmed by a software engineer – produces such a result. In order to accomplish such a performance the whole context – the situation – has to be reflected upon. That means the contradictions have to be maintained during the process of reasoning: "The wire is marked and the wire is non-marked", or in other words: "A equals non-A", and a corresponding argumentation holds for example_2.

We will leave these examples for a while (we refer to them later in our discussion) and dedicate our interest to McCulloch's citation[A1] where he writes that "the simplest surface on which this net maps topologically (without a diallel) is a tore".

In the late seventies the torus (already introduced by McCulloch in 1945) became a symbol for the so-called 'double closure' of neural processes, i.e., for systems that recursively operate not only on what they "see" but also on their operators as well (cf.[A2], [A6], [B2]). The image of the torus interpreted within this context turned into a symbol – a message – for the functional organization of living organisms, especially in the circles of second order cybernetics, socio-cybernetics, and other post or prefix derivatives of cybernetics. However, neither the symbol of a torus nor the image of networks (or more mathematically of graphs) are suitable for a theoretical explication of the term 'heterarchy' (see also the discussion in [A6]). McCulloch already gives us a hint: First there is his idea of the diallel as was already mentioned above, and second the fact that the law of transitivity can no longer be applied to the description of heterarchical structures. This has to be interpreted in terms of logic and not with images of circles, loops, networks or with Luhmann's escape into paradoxes[[15]].

From the idea of diallels one can immediately infer that heterarchical structures can never be observed in the world of physics, i.e., 'heterarchy' obviously can only be assigned to processes and not to the states of *bona fide* objects (like the table in front of us). One can further deduce that the nature of heterarchical processes cannot be attributed to the

our thinking and appear as paradoxes because physics excludes the thinking subject and any subjectivity from within the formal description.

15 Walter L. Bühl, *Luhmanns Flucht in die Paradoxie*, in: Die Logik der Systeme: Zur Kritik der systemtheoretischen Systemtheorie von Niklas Luhmann, (P.-U.Merz-Benz & G. Wagner, eds.), Universitätsverlag Konstanz, 2000, p. 225-256.

transitions between initial and final states as they occur in physics and chemistry. Physical-chemical processes – in the sense of state transitions – can be described (since the time of Newton and Leibniz) by differential equations[16] – see also [A6].

Since heterarchy and hierarchy are complementary categories of description, we can deduce that the transitivity law is valid for the (logical) description of hierarchical and heterarchical process structures but it can only be applied to hierarchical structures and not their heterarchical complements.

The transitivity law is given by the following binary relation:

$$R(x, y) \wedge R(y, z) \rightarrow R(x, z) \qquad \_1a)$$

where '$\wedge$' stands for the conjunction (AND), and '$\rightarrow$' for the implication (IF .... THEN ...)

If we assume a model of time with an ordered set of discrete time points t_1, t_2, ..., t_n , then x, y, and z can be substituted, viz.

$$x = t_1,\ y = t_2 \text{ and } z = t_3$$

Together with the relation 'smaller than' or 'earlier than' for R (symbolized by ' < ') it follows,

$$R(t_1, t_2) = (t_1 < t_2) \qquad \_1b)$$

which yields

$$[(t_1 < t_2) \wedge (t_2 < t_3)] \rightarrow (t_1 < t_3) \qquad \_2a)$$

or using the symbols of the propositional calculus

$$[(t_1 \rightarrow t_2) \wedge (t_2 \rightarrow t_3)] \rightarrow (t_1 \rightarrow t_3) \qquad \_2b)$$

If we express (2a,b) in a sentence, it reads as:

"IF t_1 is a time earlier than t_2 AND t_2 is a time earlier than t_3, THEN it can be deduced that t_1 is a time earlier than t_3."

or according to (2b)

"IF time t_1 implies t_2 AND time t_2 implies t_3, THEN it can be deduced that t_1 implies t_3."

or alternatively one can say instead of "t_1 implies t_2" that "t_1 is the predecessor of t_2" and so on.

If we take a short look at hierarchical structures, we realize that even hierarchical structures make sense only in the context of processes. For example, if we use a tree for the presentation of a hierarchical structure, then the tree is a symbol for the process which it represents.[17] Every transition from an initial state A (symbolized by a node in the tree) to a final state (node) B can be described by an ordered set of (time)points: t_1, t_2, t_3, ..., t_n for which the law of transitivity strictly holds and can therefore be applied.

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16 The concept of differential equations does not – and cannot – include the concept of diallels, and if the transitivity law cannot be applied the whole idea of time dependent differential equations is meaningless (cf.[B1]).

17 We are comparing this symbol with processes which we have experienced in the past because we are surrounded by hierarchically structured processes which can be detected and described very easily. All physical processes (transitions between two or more states) are hierarchically structured.

Now if we use the 'fishnet' definition (the graphs in figure_1c) in order to symbolize heterarchical structures as is done in the majority of publications in this field, then we can also pass from a node *A* to *B* and again the transitivity law can be applied in order to describe the time dependency of such a process, namely the transition from state *A* to state *B*. This holds even if we pass along a closed circle.[[18]] In other words, graphs or circles are inadequate symbols for the representation of heterarchical process structures. If taken as symbols for processes, graphs always represent exclusively hierarchical process structures.

All this sounds so self-evident, or even banal, that nearly no one ever questioned sequentiality as the only way to describe the time dependence of processes. However, it is this sequentiality of our concept of time which is responsible for the fact that all processes fulfill the transitivity law as far as their time dependence is concerned and therefore they are all hierarchically structured.

We are now in the position that either we have to ask whether we are to imagine non-transitive processes which we don't know yet, or that one of the fathers of cybernetics, namely McCulloch, was a poseur or even worse, an idiot. Since the latter conclusion can be excluded for many reasons[[19]], in the following discussion we have to analyze McCulloch's undiscovered hint in order to be able to imagine non-transitive processes.

As a first step on this way, we have to realize and keep in mind that the sequentiality of our concept of time is based on the use of the natural numbers where each number (except the naught) has precisely one successor and one predecessor, resulting in a 'hierarchy of values', and definitely not in a 'heterarchy of values' (cf., relation_(2)).

[18] This is no surprise because Euler (Leonhard Euler, 1707-1783) introduced the concept of graphs into mathematics in order to solve the so-called "Königsberger Brückenproblem" which he was asked to do by his friend Immanuel Kant (1724-1804).

[19] Here we recommend reading *Number and Logos* by Gotthard Günther (cf., www.vordenker.de).

2

In this section we will discuss some elementary examples of transitivity in order to understand McCulloch's definition of the 'value anomaly'. The result will be that the transitivity law is valid and can be applied for all (physical) quantities which can be measured. Non-transitivity can only be introduced into physics by laying waste to the transitivity law, viz.,

> "IF x is heavier than y AND y is faster than z, THEN it can be deduced that x is warmer than z." _3)

Needless to say that (3) can be considered a non-transitive relation. However, it is completely meaningless because the physical quantities x, y and z have different physical units, and therefore different meanings, viz.,

$$[x] = [y] = 1\text{kg} \quad \text{and} \quad [y] = [z] = \text{m/s}^2 \quad \text{and} \quad [x] = [z] = 1\text{K}$$

However, this negative example demonstrates that every physical process of measurement can be considered (from a logical point of view) an application of the transitivity law. It is used automatically in a correct way by experimentalists. The reason is quite simple: Every measurement is a comparison with a pre-defined (physical) unit, i.e., it corresponds to the specification as to whether two measurable quantities have the same measure[20]. In other words, the transitivity law can only be applied inside a given context, i.e., it does not make any sense to compare or to add apples with pears, potatoes, and crocodiles. The following example demonstrates what we have just outlined.

> "IF (temperature) T_1 *is smaller than* (temperature) T_2 AND (temperature) T_2 *is smaller than* (temperature) T_3, THEN it can be deduced that (temperature) T_1 *is smaller than* (temperature) T_3." _4a)

It is the reduction to a pre-defined context that characterizes all physical measurements. The experimentalist selects such contexts automatically. Therefore it is impossible to construct a meaningful non-transitive relation in the world of physics using physical quantities. In other words, physics is the world of quantities – a world that is characterized by a 'hierarchy of values' (in the sense of the transitivity law)[21].

The situation seems to be rather simple, if not trivial. But what about the following logical sentence:

> "IF person P prefers (the apple) *a* to (the pear) *b* AND person P prefers (the pear) *b* to (the banana) *c*, THEN it can be deduced that person P prefers (the apple) *a* to (the banana) *c*." _5a)

From a logical point of view example (5a) is correct (it is logically true) and it seems as if (5a) corresponds to the sentence given by the example of relation (4a). But is the similarity from a logical point of view really so harmonious?

The sentence given by (4a) is (logically) valid regardless whether the temperatures T_1, T_2, and T_3 have been measured from one object at different times or from different objects at

[20] The measured value, e.g., the change of a length on a thermometer can be related by the experimentalist to another quantity, for example, temperature.

[21] See for example: R. Rammal, G. Toulouse & M.A. Virasoro, *Ultrametricity for physicists*, in: Rev. Mod. Phys. 58 (1986) 765-788.

the same time or at different times. From a logical point of view relation (4a) is timeless. It is also independent of the kind of objects involved and of the location where it has been measured.

The situation of the logical sentence (5a) is quite different: First it is not clear whether the statement is valid for all kinds of apples, pears, or bananas, or whether this statement is valid for every time and/or for any possible locations. Second, apples, pears, bananas are never completely identical, even if they are from the same class of identification.[22] Third, if we consider different persons (P_1, P_2, P_3) in analogy to the sentence (4a) where we have stated that the relation's truth value is independent of whether the temperatures result from different objects or not, we can try to construct the following version of (5a):

"IF *person* P_1 prefers (the apple) *a* to (the pear) *b* AND *person* P_2 prefers (the pear) *b* to (the banana) *c*, THEN one can deduce that *person* P_3 prefers (the apple) *a* to (the banana) *c*." _5b)

There is no doubt about the nonsensical logical inference of (5b) and nobody would accept such a construction as an intelligent example of McCulloch's demand for a non-transitive 'value anomaly'. However, if we again focus our interest on some physical attributes of different persons, we are back in the world of a 'hierarchy of values'. No one will have any objections against the logical validity of the following version:

"IF (body temperature) T_1 of person P_1 *is lower than* (body temperature) T_2 of the person P_2 AND (body temperature) T_2 of person P_2 *is lower than* (body temperature) T_3 of person P_3, THEN one can deduce that (body temperature) T_1 of person P_1 is *lower than* (body temperature) T_3 of person P_3." _4b)

The idea behind these examples is to demonstrate that transitive logical statements can always be constructed for physical attributes, or physical quantities, or more generally for quantities which can be measured, and that it is difficult – if not impossible – to construct logically meaningful non-transitive sentences using physical attributes. In other words, McCulloch's 'value anomaly' or 'heterarchy of values' cannot be found in the world of physics or chemistry.

However, if attributes are involved that belong to the category of subjectivity, such as individual taste, the situation reverses and it becomes difficult to construct logically meaningful sentences where the transitivity law can be applied. For the following discussion we will therefore focus our interest on these examples in order to find a reasonable logical exemplification of McCulloch's 'value anomaly'.

In order to simplify the discussion that follows we assume at the outset that all apples, pears, and bananas are identical, i.e., we do not differentiate between different sorts of apples, pears, and bananas. Yet even this simplification does not give us any information as to whether the logical statement (5a) holds at all times. In other words, we can expect that the criteria for the decisions of person P are certainly not invariant, i.e., they are changing from time to time. This causes the main difference between (5a) and (4a,b), and we therefore cautiously speculate that we will find the key for a better understanding of heterarchical, i.e., non-transitive structures if we further investigate these examples. In order to analyze this, we will make a short excursion into the world of modal logic.

22 The problem of identity for a formal mathematical description of living systems has been discussed by Walter M. Elsasser, *A Form of Logic Suited for Biology*, published in: "Progress in Theoretical Biology", (Robert Rosen, ed.), Volume 6, p.23-62, Academic Press, 1981 (cf., [A3])

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In 1945 McCulloch fortunately could know nothing about modal logic (cf., [C1]); modal logic might have obscured his view of the essentials of the problem. We will explain this in the following. As discussed above, the criteria for a decision of person P concerning the preference of different fruits will change in time. If we introduce different *situations* (or different *possible worlds*) which we call s_1, s_2, s_3, ..., s_n and if we assign the statements concerning the preferences of *a*, *b*, and *c* of the person P in (5a) to three different situations we get the following expression.

s_1 : $[(a \succ b) \wedge (b \succ c)] \rightarrow (a \succ c)$ _6a)

s_2 : $[(a \succ b) \wedge (b \succ c)] \rightarrow (a \succ c)$ _6b)

s_3 : $[(a \prec b) \wedge (b \prec c)] \rightarrow (a \prec c)$ _6c)

⋮ ⋮

Instead of (6c) the following relation can be written:

s_3 : $[(b \succ a) \wedge (c \succ b)] \rightarrow (c \succ a)$ _6c)

The symbol ' $\succ$ ' stands for the binary relation of (5a,b):

"*person* P *prefers* ... *to* ..."

The relation (6a,b) reads as follows:

> "IF person P prefers in situation s_1(or s_2) *a* to *b* AND person P prefers in situation s_1(or s_2) *b* to *c*, THEN it can be deduced that person P prefers in situation s_1(or s_2) *a* to *c*."

and relation (6c) reads:

> "IF person P prefers in situation s_3 *b* to *a* AND person P prefers in situation s_3 *c* to *b*, THEN it can be deduced that person P prefers in situation s_3 *c* to *a*."

Now we speculate and introduce a possible modal logical interpretation of non-transitivity:

> *Since in 1945 modal logic was unknown to McCulloch, he may have had the model of 'many possible worlds' in mind when he wrote his paper, and therefore the idea concerning the 'value anomaly', non-transitivity, could be interpreted by the relation which results if the terms marked in red in (6a-c) are considered, viz.,*
>
> $[(a \succ b) \wedge (b \succ c)] \rightarrow (a \prec c)$ _7a)

If we translate (7a) into a written English sentence, it reads precisely as what is in McCulloch's paper from 1945 (cf., [A1]):

> "Consider the case of three choices, *a* or *b*, *b* or *c*, and *a* or *c* in which *a* is preferred to *b*, *b* to *c*, and *c* to *a*."

We certainly do not know what McCulloch had in mind when he published *A heterarchy of values...*, therefore we are left on our own and have to use our own brains in order to interpret his intellectual inheritance. The question is whether we get an idea of the 'value anomaly', the 'heterarchy of values' in the sense of non-transitivity using the relations (6a-c) and (7a). There is no doubt that the form of relation (7a) has no logical meaning within the model of many possible worlds, i.e., within modal logic. If we introduce indices in order to distinguish different situations, then it follows:

$$[(a \succ b)_{s1} \wedge (b \succ c)_{s2}] \rightarrow (a \prec c)_{s3} \qquad \_7b)$$

Although such an indexing as it is given in (7b) is not really defined within the concept of modal logic it still gives us a hint. We could start to interpret the different situations in (7b) by different preferences of three persons. But this is an intellectual dead end as far as our goal is concerned, namely to find a reasonable explication of non-transitive process structures. Since relation (7) yields some kind of non-transitive structure of thinking we will continue to focus our interest on this relation.

If we relate (7) to a single person, then the different situations s_1, s_2, and s_3 can only be interpreted correctly within modal logic if we introduce different time points, i.e, if we relate the different situations to different times:

$$[(a \succ b) \wedge (b \succ c)]_{s1(:=t1)} \wedge [(b \succ a) \wedge (c \succ b)]_{s3(:=t3)}$$

"person P prefers in situation s_1 (at time t_1) an apple *a* to pear *b* AND pear *b* to banana *c* AND in situation s_3 (at time t_3) pear *b* to apple *a* AND banana *c* to the pear *b*" _8b)[23]

This is an interpretation which is correct within the linguistic framework of modal logic. But such an interpretation gives us a transitive relation (with respect to the time points) resulting in a 'hierarchy of values' and again, we have missed our goal.[24]

If we relate (7) to a single person but without considering different points in time, then the following interpretation results:

"person P prefers (apple) *a* to (pear) *b* AND (pear) *b* to (banana) *c* AND (pear) *b* to (apple) *a* AND (banana) *c* to (pear) *b*"

which can be expressed as

$$[(a \succ b) \wedge (b \succ c)] \wedge [(b \succ a) \wedge (c \succ b)] \qquad \_8a)[23]$$

While (8b) represents a correct interpretation of relation (7) we lost the non-transitive aspect whereas relation (8a) has no logical meaning at all. It looks like as if modal logic is not an adequate logical tool to model a 'heterarchy of values'. We will come back later in our discussion to the limitations of the modal logic and there we will give a few more general arguments why modal logic and other non-standard logics are incompatible with McCulloch's demand for a 'value anomaly', i.e., a 'heterarchy of values'.

23 The number of the relation was chosen in order to correspond to the German version of the text.
24 cf. relation (2) and the side-step contribution_[C1] concerning modal logic and time logic.

4

In the following we keep focused on relation (7) but leave the world of modal logic behind. We assume that different situations s_1, s_2, ..., s_n may now be interpreted as different contexts. For our fruit example this could be the maturity rate, the nature of fruit growing, the region where the fruit was grown, the prices or the individual taste of **the person who has to make a personal decision** as to which of the different kinds of fruit he will buy at a market.

It is important to realize that any relation represented by (8b) makes sense only if the person already has reached its decision. The process of decision itself cannot be described by relations like (8b). Every decision process is a process with an open end, a process which is not determined. For our fruit example this means that the person also could decide to buy no fruits at all and buy sandwiches instead, i.e., the person rejects the whole situation and its alternatives given by s_1 to s_n in our fruit example.

In other words, every decision process refuses – as a matter of principle – any positive-linguistic logical formulation. The reason is very simple, only an already given decision – the result of the decision process – can be expressed in a positive-linguistic logical formulation but not the ongoing process itself in which a deciding person alone is involved (see also [A6]).

Obviously it is the 'process of decision' itself that has to be analyzed to understand what distinguishes a 'heterarchy of values' from a 'hierarchy of values' from a logical point of view. In other words, the decision process – as a process(!) – cannot be described within the framework of positive-linguistic formal languages, and therefore its structure cannot be hierarchical. At the moment we will give only a brief argument to support this assertion: At the end of any decision process there has to be a designation in order to make the decision. The designation corresponds to a selection of (a declaration of ..., a decision for ...) a position, whereas a non-designation corresponds to a negation of possible positions.[[25]] The process of designation can be described by standard logic as well as by all known so-called non-standard logical calculi like modal logic since they are truth-definite calculi, and therefore positive-linguistic formal tools of thinking.

Later in our discussion we will come back to this point. At the current stage of discussion a more precise definition cannot be given, since we have not yet introduced the concept of a negative language which is necessary to formulate a precise definition of the complementary relation between negative and positive languages. The situation corresponds strongly to the relation of complementarity of 'heterarchy and hierarchy'.

If we consider once again the relations (4), (6a-c), (7b) and (8b) [A5] we have shown that relation (4) represents a logical sentence which is time-independent, i.e. which is timeless. This does not mean that the temperature of an object may not change after it has been measured – this is not the point. It is the logical statement given by relation (4) that is of

[25] In an over-simplified approach one could say that the designation corresponds to the selection of the logical truth-value and the non-designation to the selection of a negation. While the latter is a nearly correct association, the former (concerning the truth-values) is not. This circumstance will be explained later in our discussion.

importance; this statement itself is timeless. In general, logic and mathematics are timeless, i.e., they are tools of thinking that are sufficient to describe (physical) states and transitions between (physical) states, but not mental processes as we will discuss below. State transitions can be measured[26] and therefore they can be (logically) designated and described within a positive-linguistic framework of classical logic and mathematics (cf., [B1]). The physical parameter time as an order parameter in the sense of the transitivity law also can be designated and represented in a positive-linguistic formal language. These (positive-linguistic) categories of thinking are well suited for the description of our physical world — the so-called *bona fide* objects. Relation (4) exemplifies this feature from a logical point of view. What cannot be measured in a physical sense are mental processes such as 'reasoning', 'deciding', 'perceiving', 'learning', etc. which are processes and not states or transitions between states in a physical sense. And it is this type of processuality which was envisaged by McCulloch more than fifty years ago[27] in *A heterarchy of values...* (cf. [A6]).

What's in the brain that ink may character? This is the title of an essay published by McCulloch in 1964.[28]. Today we can answer this question carefully. The negative answer first: Mental processes can never be modeled within a positive-linguistic framework of language; therefore they cannot be written down either by ink or by a word-processing program. For the positive version of the answer we return to our fruit example and the decision-making process of our person P. What we are able to say is that within the brain of our person P different situations, different standpoints have to be considered in order to come to a decision:

> Required for the further discussion is a standpoint-dependent consideration of the decision process.

From a logical point of view every standpoint has to be characterized by at least one logical domain[29] – in a manner of speaking it is the logical place from which a standpoint can be presented. If we consider the different situations of relation (6) as different logical places $s_1, s_2, \ldots, s_n$ then each of these places should be characterized by (at least) one logical domain, otherwise we cannot speak of a logical place. Instead of one logic as is the case in the many possible worlds model, modal logic, we now have (at least) as many possible logical domains as there are possible worlds. However for modeling a decision process this is not enough; the different logical places, i.e., the logical domains have to be mediated by some (adequate) logical operators. The model of many possible worlds, modal logic, is characterized by just one logic or one logical domain; correspondingly only one standpoint exists, i.e., there are many possible worlds but only one logic, one standpoint – that's all that modal logic is able to deliver. And if

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26 For the definitions of a physical state and a physical system as well as for the transitions between physical states, see for example [B1]

27 Note: McCulloch's paper was written in 1945 in English(!) and today is still not understood (nearly 60 years after its publication) by the English speaking readers. This is a period of time that corresponds nearly to the length of a human generation – a shameful situation!

28 W.S.McCulloch, *What's in the brain that ink may character?*, in: Embodiments of Mind, MIT Press, 1970, p.387-397.

29 Note: Actually more than one logical domain for each standpoint is needed. However, for reasons of simplicity we will not consider this point at the moment.

there is only one standpoint, consequently, there is no need for a mediation of different standpoints.

<u>Result</u>: The model of the modal logic does not deliver a formal basis for any standpoint-dependent theory.

The situation has been illustrated graphically in figure_3[C2]: Figure_3a depicts the situation of modal logic, and in figure_3b there are three logical systems, three logical domains which are not mediated but instead isolated from each other, i.e., there are no special operators that allow transitions between the different logical domains. Later in our discussion we will come back to this point.

* * *

For an example of a standpoint-dependent situation we will discuss the case of the marked unmarked two-wired cable which is shown in figure_2. First the cable can be considered from the standpoint in which both wires are without any markers (flags), i.e., one wire can be marked, for example, by a colored flag. From a second point of view both wires are marked, for example, with red flags, i.e., in this situation one marker can be removed in order to distinguish both wires. Further standpoints are given by the situation in which one of the two wires is marked, and so on. In order to model the decision process through comparing the different standpoints and their mutual relationship in a formal mathematical way, it is not enough to consider the different situations sequentially, i.e., one after the other, because this already produces an ordered relation between the different standpoints – a hierarchy of values – which requires a decision to have already been made. This, however, contradicts the assigned task. What is needed is a process model which guarantees the equivalence of the standpoints, the logical domains, i.e., the decision process has to be a parallel simultaneous complex of a multitude of non-transitive processes – a processuality which we are used to calling a decision-making process. This can only be achieved by a 'heterarchy of values', i.e., by heterarchically structured processuality.[30] Any super- or sub-ordination of different standpoint can only occur if a final decision has been reached, i.e., at the end of the decision-making process. In other words it is the parallel simultaneous mediation of the different standpoints that enables our brains to maintain (dialectical) contradictions until the dialectical contradictions are sublated[31] and a decision has been reached – a decision which possibly could be a rejection of the whole situation.

A similar consideration can be made for all the other examples (the sentence which contains three errors, or the decision process in our fruit example).

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30 <u>Note</u>: We use the parallel simultaneity of processes and heterarchically structured processes as synonymous concepts.

31 <u>Note</u>:

a) In the example with the two wires the decision could either be that the wire with the flag is designated as marked or the decision could be that the wire without the flag is designated as marked – we are back, of course, in the world of the classical logic.

b) Aufhebung (sublation): Sublation, a translation of the German word Aufhebung, is a key concept of dialectics. In his Philosophical Notebooks, Lenin defines sublation as "terminate but simultaneously preserve". In Hegel's dialectic, when contradictory positions are reconciled in a higher unity (synthesis) they are both annulled and preserved (aufgehoben).

5

There is a common feature that characterizes all decision processes: They are cognitive-volitive processes. The accent lies on 'cognitive and volitive' and on 'processes' (cf., [C4]).

It is definitely not possible to separate cognition from volition and *vice versa* as is suggested by Maturana's[[32]] definitions of the living[[33]] and as was even applied to the description of social systems by Niklas Luhmann and his epigones in soulful circular harmony but without any critical reflection. Cognition and volition are inseparable processes, i.e. they exist as a parallel-simultaneously mediated co-existing processuality (see also: [C4]). This feature will be discussed further in the example of character recognition (cf., figure_4 [C3]) in the context of the standpoint-dependency of a cognitive-volitive processuality.

The following example of character recognition was not presented here in order to develop a new software concept, but to illustrate the principles of a cognitive-volitive process (cf., legend of figure_4). The software used today has nothing in common with a model of a cognitive-volitive process, i.e., the computer recognizes nothing and still does not make any decision. In other words, contemporary software does not model standpoint dependencies. The question here is what is required in order to model standpoint dependencies. For this reason we have chosen the example of character recognition.

In the legend of figure_4[C3] we have already pointed to the fact that at least four logical places (standpoints), i.e., four mediated 2-valued[[34]] logical domains, are necessary in order to generate a logical system which allows for modeling the smallest amount of standpoint dependency.

The fourth logical place is necessary in order to reject the whole situation.[[35]] This possibility already demonstrates that more than four logical places are necessary in general. This supposition results mainly from the fact that a rejection of a complete

[32] "... Living systems are cognitive systems, and life as a process is a process of cognition. This statement is valid for all organisms, with and without a nervous system...
...The nervous system expands the cognitive domain of the living system by making possible interactions with 'pure relations' ; it does not create cognition..."
See for example:
H. R. Maturana, *Biology of Cognition*, in: *Autopoiesis and Cognition – The realization of the living*, (H.R. Maturana & F.J. Varela, eds.), D. Reidel Publ., 1980.

[33] Note: From a biological point of view Maturana's definition is acceptable because as a biologist he also had to consider plants as living systems. For plants it is already difficult (but possible) to discover and to explain cognition but it is not quite clear with volition. However, so far as animals or even human beings are concerned, it is not enough to reduce the definition of a living system to its cognitive abilities, and it becomes unacceptable if not ridiculous if such a reduction is done in social sciences.

[34] Note: For three- or higher valued logical systems the number of domains differs, but this will not be further discussed in the present investigation.

[35] Note: Günther's logic of place-values (as well as the polycontextural logic) starts (as a logical system) with 4 (four) place-values and not with 3 (three) as it is the case in Peirce' experiments of a Triadic logic. Gotthard Günther and later Rudolf Kaehr both have shown that with 3 place-values the system is not a complete logical system.

situation (in the present example it is the situation of character recognition) is only meaningful in front of background-"knowledge". However, the background-"knowledge" has to be completed or even generated first during the process of character-recognition. This has to be expected from an intelligent technical system with the ability for cognition and volition. In other words, additional contexts, i.e., additional logical places, additional standpoints are necessary for any successful implementation.

Figure_4[C3] shows the word 'Fall' from which the character 'a' (which has to be recognized) is taken. The word 'Fall' occurs in a footnote of a letter. The language of the letter is German, the content of the letter is a logical discourse, and so forth (cf., figure_5a,b[A4]) In other words, there are several further contexts which first have to be discovered by an intelligent system, and second have to be considered for the interpretation of the characters, and finally of the written text.

We have listed the different relations for reasons of completeness and also to point to the fact that the problem of automatic (con)text processing is – from a theoretical point of view – similar to a simple process of character recognition if standpoint dependency is considered.

**How can we imagine the mediation of different logical places,
of different standpoints?**

6

In order to answer the question given above, we turn back to figure_3[C2]. If from the camp of Artificial Intelligence there is talk of 'multimodal reasoning' this foretells the beginning of the search for and the design of standpoint dependent process models where different (non-mediated) meta-levels like in figure_3b are taken to be the central theme. John Sowa, for example, writes[[36]]:

> "Unfortunately, Kripke's model structures lead to a combinatorial explosion when they are extended to all the varieties of modality and intentionality that people routinely use in ordinary language....
> ... this paper [John F. Sowa, *Laws, Facts, and Contexts*] defines a family of nested graph models, which can be specialized to a wide variety of model structures, including Kripke's models, situation semantics, temporal models, and many variations of them. An important advantage of nested graph models is the option of partitioning the reasoning tasks into separate metalevel stages, each of which can be axiomatized in classical first-order logic..."

On the basis of these hierarchically structured models it is as a matter of principle impossible to model cognitive-volitive processes or to implement such process models. This fact was seen precisely by McCulloch nearly 60 years ago.

Schematically figure_6[D1] shows a logical system with three mediated logical domains or logical places. In the following we will speak of logical **contextures** instead of logical domains or logical places. The complex of three mediated contextures L_1, L_2 and L_3 is the smallest irreducible unit possible. Gotthard Günther, who introduced the terms contexture and polycontexturality (see below) into science, called this complex of mediated contextures the **proemial-realationship**.[[37]] The representation in figure_6b[D1] is chosen in order to denote that this is a structure which allows the modeling of different standpoints. What is logically true from one point of view may be logically false from another point of view.

In order to demonstrate how a mediation of different contextures (standpoints) can take place, we will discuss the situation given by (7a,b)[[38]] within the context of a (cognitive-volitive) decision process and with the help of the proemial-relationship of figure_6[D1].

During the course of our discussion it has turned out that a decision process is a cognitive-volitive process where a non-transitive relation like the one given by the sentence (7) has to be postulated. As we have demonstrated above, relation (7) cannot be interpreted reasonably within the linguistic framework of truth-definite standard logic and/or non-standard logic such as modal logic. Our problem then turned to the point that McCulloch's

[36] John F. Sowa, *Laws, Facts, and Contexts: Foundations for Multimodal Reasoning*, in: Knowledge Contributions, V. F. Hendricks, K. F. Jørgensen, S. A. Pedersen (eds.) Kluwer Academic Publishers, 2003.
Note: The term "Kripke's model structures" denotes the model of many possible worlds, which we have introduced as modal logic.

[37] See for example ref. 7 : proemial relationship (Greek: prooimion = prelude)

[38] Within the side-step contribution_[A5] a collection of all formulas of the present article is given in order to facilitate the reading.

'heterarchy of values', i.e., the non-transitivity could not be justified in a rational logical way.

Therefore in what follows we consider relation (7) within the linguistic framework of the theory of polycontexturality and we use Günther's logic of place-values. Although this is still a semi-classical approach and not really a polycontextural one, it nevertheless gives us some notion as to the answer and a precise logical definition of the nature of a heterarchically-structured processuality.

Table_[D3] collects some basic logical operations like conjunction, disjunction, negation, and implication for Günther's logic of place-values in order to facilitate what follows.

Most of the following tables are taken form *Cognition and Volition*[39]and from *Again Computers and the Brain* [40]. In the latter a slightly different notation is used: Instead of numbers for the description of place-values the following symbols are used in order to mark different place-values, i.e., standpoints:

$$\{T_1, T_3\} := T_{1,3}; \qquad \{F_1, T_2\} := F_{1,2}; \qquad \{F_2, F_3\} := \mathsf{F}_{2,3} \qquad \_9)$$

The different subscripts are related to the corresponding logical places (place-values); they can also be related to the different contextures or standpoints. This representation has been chosen in order to make clear that it is empty nonsense to speak of different "truth-values" in Günther's logic of place-values as has been done in the past by some reviewers of Günther's essays (cf., [D2]). Different "truth-values" cannot exist because an object can only be designated once: a rose is a rose is a rose.... and an electron is an electron is an electron, ...

From the fact that an object can be designated only once and that several different "truth-values" can never exist, one cannot deduce that only one negation exists like it might be suggested from classical logic. If one accepts different standpoints (different situations, different worlds, etc.) within a logical discourse then only one standpoint can be designated. But in principle it is possible to negate all of the others, i.e., the rest is non-designated. In order to clarify this position, which results from the assumption that many logical places (standpoints) are accepted, two different symbols F, F (false, or better: non-designated) and only one symbol T (true, or better: designated) are introduced in *Again Computers*... In order to relate these symbols to the place-values they are also shown in figure_7[D4]. For our further discussion we will not use the T, F, F-symbolism.

In the occupancy table[41] of figure_7 the positions of a mediation between the three contextures are marked by connecting lines in order to highlight the relation between figure_7 and figure_6.

How can the symbols in table 7[D4] be interpreted ?

[39] see ref. 7

[40] see ref. 8

[41] Note: It is an occupancy table and not a truth-table. The place-values have nothing to do with true and false in the classical sense. They stand for 'designated' and for 'non-designated'.

The numbers symbolize place-values (or standpoints) as is mentioned above. If 1 is declared as 'position' (position 1 has been designated) then the numbers 2 and 3 and so forth are interpreted as 'negation' (in relation to position 1), i.e. they are non-designated. This is an interpretation from within the framework of Günther's logic of place-values, an approach which Günther very often used in his work. As we have already mentioned, the logic of place-values denotes the border between classical standard and non-standard logics on one side and the trans-classical polycontextural logic on the other side. Therefore the logic of place-values can be considered a semi-classical (or semi-transclassical) calculus. Since it was Gotthard Günther who generated the borderline between classical and trans-classical logic it is quite natural that he moved very often back and forth along this line.

Another and more general interpretation of the ciphers in table 7a[D4] results if the ciphers are considered as a part of a pattern, a so-called morphogram. Here the totality of the ciphers in a column constitute a pattern which is determined by the relative position of the ciphers. In order to visualize this aspect, symbols have been introduced in part (c) of table 7 which do not have any value like the symbol of a cipher. Günther introduced these symbols as keno-signs (kenos (ancient Greek) = emptiness, void). The complete sequence of kenos (this corresponds to a column in figure_7c) is called a morphogram. In order to give a brief impression of the meaning of morphograms, some have been depicted in figure_8 and _9[D6].

The morphograms can be used in order to index different contextures (instead of using place-values); they also can be interpreted as numbers - so-called keno-numbers. It is possible, of course, to calculate with keno-numbers as well. The rules for the relation of contextures, keno-sequences, and morphograms with each other as well as for calculation with keno-numbers are a part of keno- and morphogrammatics and the theory of qualitative numbers – a 'heterarchy of values'.[42]

[42] a) R. Kaehr, *Materialien zur Formalisierung der dialektischen Logik und Morphogrammatik 1973-1975*, in: Gotthard Günther, Idee und Grundriß einer nicht-Arsitotelischen Logik, Felix Meiner Verlag, Hamburg, ^{2}179.
b) E. Kronthaler, *Grundlegung einer Mathematik der Qualitäten – Zahl-Zeichen-Spur-Tao*, Peter Lang Verlag, Frankfurt, 1986.
c) R. Kaehr & Th. Mahler, *Morphogrammatik – Eine Einführung in die Theorie der Form*, in: Klagenfurter Beiträge zur Technikdiskussion, Heft 65, 1993.
d) Siehe auch: Rudolf Kaehr, *KOMPASS - Expositionen und Programmatische Hinweise zur weiteren Lektüre der Schriften Gotthard Günthers*: http://www.vordenker.de
e) R. Kaehr & J. Ditterich, E*inübung in eine andere Lektüre: Diagramm einer Rekonstruktion der Güntherschen Theorie der Negativsprachen*, Philosophisches Jahrbuch, 86. Jhg., 1979, S. 385-408. (siehe auch www.vordenker.de).

7

Before we return to the starting point of our discussion, namely trying to understand the relation between heterarchy and hierarchy in logical terms, we will shortly summarize some of the main points we have discussed so far.

From the argumentation given in connection with relation (7)[43] it should have become clear that within the linguistic framework of modal logic, i.e., the model of many possible worlds, McCulloch's demand for a 'heterarchy of values' cannot be modeled in a formal-mathematical way. The reason for this shortfall can be given: modal logic is, like all other derivatives of standard logic, a truth-definite logic. Such a logic is suited for the description of states and transitions between states as they occur in physics, i.e., something is or it is not. In other words, something can be designated (but only once!) or it cannot be designated, but then there is nothing – a Third is excluded(!). In terms of the Aristotelian axioms, this can be expressed as:

> A physical quantity *A* is or is not (law of identity), it cannot be both '*A* AND non-*A*' (law of contradiction) and *A* cannot be something else, a Third (law of excluded middle, *tertium non datur*): (a_7)
>
> An electron is an electron, is an electron, is an electron, ..., and not a non-electron, e.g., a voltage, or a rose, or a whatsoever.

Therefore a truth-definite logic is not suited for modeling a processuality which describes different standpoints (different logical places) and which is characterized by a parallel simultaneity of processes, as was already mentioned above. All mental processes – for instance learning, reasoning, decision-making, etc. – belong to this category of processes.[44]

This sounds surprising because we have all learned that the transitions between different states as they occur in the world of physics and chemistry, which we call 'processes', can be described mathematically with differential equations. This idea of a sequentiality of points in time, which basically determines the concept of differential calculus[45], has been so successful that it is applied as a tool of description for all kinds of processes in economics, sociology, biology, medicine, and last but not least in the field of brain research. It just seems as if this idea of a sequential processuality is given by God as an irrefutable reality that cannot be challenged because we have no other experiences and we cannot measure any other processuality – this turns out to be an intellectual problem, at least in the Western world, where thinking is dominated by natural sciences. It looks as if today only measurable things are considered scientific reality – an immediate result of our positive-linguistic scientific language/logic. This is the neo-positivistic view of the world: A (physical) state is or it is not and if a state is, then it is possible to describe the transitions from an initial state to a final state as an ordered sequence of transition states using the tools of a timeless mathematics (cf., [B1]).

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43 The attached file_[A5] contains a collection of all formulas of the present article.

44 Today the neural networks are very often cited as a model for learning processes. The neural network models are adaptive data filters and have nothing to do with real learning process – cf. ref. 8.

45 Both Newton (Sir Isaac Newton, 1643-1727) and in parallel and independently Leibniz (Gottfried Wilhelm Freiherr von Leibniz, 1646-1716) invented differential calculus in order to model and to describe the movements of physical bodies.

So much for our short summary. But what about contemporary computers?

Our computers work on the basis, i.e., on the assumption, that all algorithms are given by a sequence of instructions:

> All algorithms known today describe processes only in the sense of state transitions, i.e., all algorithms can be represented by a sequence of ordered time points independent of what programming language will be used for their implementation (cf., [B3]).

This is the reason why all known algorithms can be represented within the function model of the Turing machine(cf., [B4]), a result which seems to be so evident, that it is even used for the definition of an algorithm as a 'Turing computable function'.

The functionality of a Turing machine is a strict sequentiality, i.e., all (arithmetic) operations are executed in a (time)sequence of single steps where the transitivity law strictly holds (cf., [B4]). In other words: On the basis of the (Church-)Turing paradigm only hierarchically structured algorithms/processes can be modeled and implemented. Heterarchically structured algorithms/processes belong to a non-Turing world, i.e., they do not exist within the world of Turing machines, which is also the world of our contemporary computers – this is one of the basic and underlying problems of all conceptions developed by recent Artificial Intelligence research.[[46]]

For a (non-Turing)computer world with mediated contextures as shown symbolically in figure_6[D1] we have to define some terms which do not exist in the world of the Turing machines, a world with only one contexture:

> All transitions/processes within one contexture, i.e., intra-contextural, are hierarchically structured while all inter-contextural transitions/processes, i.e., transitions/processes between different contextures are heterarchically structured.
>
> In a certain way this is the definition of 'heterarchy' within the linguistic-framework of the polycontextural theory.

- ### Justification of the given definition of 'heterarchy':

  Within one contexture – intra-contextural – the transitivity law is valid and applicable. This is the world of physics or the computer world of the Turing-Church thesis.

  The inter-contextural processuality is non-transitive. The transitivity law cannot be applied, because it is only defined intra-contexturally – within a contexture – and not for transitions between contextures. Therefore the inter-contextural transitions/processes cannot be described in the sense of classical sequential time steps.

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[46] See also:
a) E. von Goldammer, *Zeit-Mehrzeitigkeit-Polyrhythmie* oder *das polylogische orchestrion*, in: Theorie – Prozess – Selbstreferenz, (Oliver Jahraus & Nina Ort, Hrsg.), UVK-Verlagsgesellschaft, Konstanz, 2003, p.129-185.
b) E. von Goldammer, *Betrachtungen über eine bekannte Unbekannte: Die Zeit*, in: www.vordenker.de

These processes/transitions represent McCulloch's co-ordinated, i.e., heterarchical processes (see also [A6]).

Since "pure" inter-contextural processes cannot exist like pure substances which can be isolated or hierarchical processes – physical processes – which can be observed, it follows that all mental processes have to be represented or modeled as an (dialectical) interplay of heterarchical and hierarchical processes. This can be described in a somewhat simplified way by using our fruit example of a decision process as follows: During the decision process a designation of a standpoint (a contexture or a compound contexture[[47]]), i.e., a decision for something will be made from which an action results. Actions can always be described mono-contexturally (better: intra-contexturally, i.e. within a contexture) – this again is the world of physics and chemistry, the world of measurable quantities, the world of the contemporary computers, and the intellectual framework of positivism.

The process of decision itself, i.e., before a designation occurs, is essentially determined by inter-contextural, i.e., non-designative processes/transitions. Since any decision as a whole is a process or better a processuality (i.e., many parallel, simultaneously running processes are involved) and not a state, non-designation means a negation of something related to something else, i.e., a negation of a standpoint (a contexture or compound contexture) which only makes sense in relation to other standpoints.[[48]] In this way a chain of negations or cycles of negations are produced for which Günther introduced the term 'negative language' (Negativsprache)[[49]] This is the potential world of qualities; it is the scientific world where subjectivity is included and not *a priori* excluded as happens in the mono-contextural world of the natural sciences.

It is very important to realize that cognition <u>and</u> volition cannot be considered separately as may be suggested by the somewhat oversimplified consideration given above. However any verbal (semantic) description of these processuality must necessarily be an oversimplification because our language and our writing always occurs sequentially!

One can state that cognition and volition have to be considered a inseparable interwoven netting, a dialectical interplay of a heterarchical and hierarchical processuality (see also [C4]).

***

[47] A compound contexture which is a combination of several contextures is possible if instead of the place values the keno-numbers are introduced for a subscription of the contextures. Within the logic of place values the idea of a compound contexture does not make sense.

[48] <u>Note</u>: a 'negation of something that is related to something else' does not exist in a positive-linguistic (truth-valued) logic. An example: 'an electron is' means there is something which we call an 'electron': "an electron is an electron". If we call the sentence as A, viz.:
'an electron is' = A, then non-A means 'an electron is not' or "it is not true, that an electron is an electron", but what is it? – answer: "nothing", "nothingness", because any Third is excluded by the *tertium non datur*. In other words, there cannot be any relation to something else, i.e. to another standpoint, another logical place. – If we take the example of the 2-wired cable, the situation is completely different. To negate one of several standpoints means that we also have to consider all the other mediated standpoints, because they are related to each other (cf., fig_7[D4], [D5]).

[49] Gotthard Günther, Identität, Gegenidentität und Negativsprache, Lecture: Internationaler Hegel-Kongreß, Belgrad 1979. published in: Hegeljahrbücher 1979, p.22-88. See also: www.vordenker.de

Upshot: A heterarchical process structure is determined by the inter-contextural transitions – so-called discontexturalities. Inter-contextural transitions cannot be represented within a positive-linguistic framework of language. Therefore the complementary term 'negative-language' was introduced in 1979 by Gotthard Günther, who was also the first to interpret McCullochs *A heterachy of values ...* logically in a series of several essays in the seventies.[[50]]

If we substitute the symbol ' $\succ$ ' in relation (7) (cf., [A5]) by the symbol for the implication ' $\rightarrow$ ' then (7b) can be written as:

$$[(a \rightarrow b)_{s1} \wedge (b \rightarrow c)_{s2}] \rightarrow (a \leftarrow c)_{s3} \qquad \_7b)$$

Within a 3-contextural representation $L^{(3)} = (L_1, L_2, L_3)$ one yields:

$$L^{(3)}: \begin{cases} L_1 & (a_1 \rightarrow b_1) \wedge (b_1 \rightarrow c_1) \rightarrow (a_1 \rightarrow c_1) \\ L_2 & (a_2 \rightarrow b_2) \wedge (b_2 \rightarrow c_2) \rightarrow (a_2 \rightarrow c_2) \\ L_3 & (a_3 \leftarrow b_3) \wedge (b_3 \leftarrow c_3) \rightarrow (a_3 \leftarrow c_3) \end{cases} \qquad \_10a)$$

Relation (10a) can be written in a short notation, viz.,

$$L^{(3)}: \quad (a \rightarrow\rightarrow\leftarrow b) \wedge\wedge\wedge (b \rightarrow\rightarrow\leftarrow c) \rightarrow\rightarrow\rightarrow (a \rightarrow\rightarrow\leftarrow c) \qquad \_10b)$$

Relation (10) symbolizes a compound of three mediated contextures, i.e., logical operations can be carried out between the different contextures (or standpoints or logical places). In the polycontextural theory these operations are primarily negations. In the semi-classical logic of place-values Günther also uses conjunctions, disjunctions, and implications which are always attributed to different place-values. The different place-values indicate a contexture (or standpoint, or place-value) – cf. figure_7[D4]. The tables in figure_10[D5] depict further logical connectives with four standpoints/place-values. These tables are easily constructable using the table 'basic operations'[D3] together with table (7)[D4].

What is important in the present discussion can already be seen from relation (10) in connection with (7b)[A5]. While modal logic does not allow logical operations between – or better, with the different possible worlds s_1, s_2, ..., polycontextural logic as well as the logic of place-values allows for a chain of implications in the following manner:

$$(a_1 \rightarrow b_1) \wedge (a_2 \rightarrow b_2) \rightarrow (a_3 \leftarrow b_3)$$

where the indices are precisely defined as markers of the corresponding logical place-values.

Such a relation is not defined in Kripke's multiverse (modal logic) for reasons that are very simple: although there are many possible worlds (or situations) in Kripke's

[50] Cf., Gotthard Günther: *Cognition and Volition* (ref.7),
--- *Identität-Gegenidentität, Negativsprache* (ref.49),
--- *Janusgesicht der Dialektik*, in: G. Günther, Beiträge zur Grundlegung einer opertionsfähigen Dialektik, Band 2, Felix Meiner, Hamburg 1979, p. 307-335.
--- Life as Polycontexturality, in: G. Günther, Beiträge zur Grundlegung einer opertionsfähigen Dialektik, Band 2, Felix Meiner, Hamburg 1979, p. 283-306.

multiverse, there is only one logic, i.e., only one contexture. Modal logic is like all the rest of classical non-standard logics – mono-contextural.

* * *

Finally we will focus on our problem from a slightly different point of view which is well-known to anyone who was already concerned with logic. The following examples are characteristic of a series of analogous cases which are of fundamental importance, e.g., in the research of artificial reasoning.

We consider again our fruit example (5)

> "IF person P prefers (apple) *a* to (pear) *b* AND person P prefers (pear) *b* to (banana) *c*, THEN it can be deduced that person P prefers (apple) *a* to (banana) *c*."

which now will be discussed from the point of view of a businessman. We use the so-called chain syllogism as an inference rule, viz.,

$$\frac{a \rightarrow b \quad b \rightarrow c}{a \rightarrow c}$$

where the logical variables stand for

a := *the apples will be harvested and offered for sale before their maturity.*	b := *the pears are preferred by the customer.* c := *the apples decompose.*

From the chain syllogism it follows:

IF the apples will be harvested and offered for sale before their maturity, THEN the pears are preferred by the customer.	$(a \rightarrow b) =_{def} A$	
IF the pears are preferred by the customer, THEN the apples decompose.	$(b \rightarrow c) =_{def} B$	_11a)
conclusion: IF the apples will be harvested and offered for sale before their maturity, THEN the apples decompose.	$(a \rightarrow c) =_{def} C$	

Written in the usual notation of the propositional calculus (11a):

$(a \rightarrow b) \wedge (b \rightarrow c) \rightarrow (a \rightarrow c)$:= syntactically always true (tautology) _11b)

One can easily convince oneself that (11) is logically true for all combinations of occupancy with the logical values w (true) and f (false) for all possible combinations of the variables *a*, *b*, and *c*. From a syntactical point of view relation (11) is a tautology as occurs for every syllogism. Table (12) (column 8) [B5] depicts the corresponding result.

Here is a further example, using the conjunctive syllogism:

$$\frac{a \rightarrow c}{a \wedge b \rightarrow c}$$

a figure which can be re-written in the usual notation of the propositional calculus:

$$(a \rightarrow c) \rightarrow ((a \wedge b) \rightarrow c)$$

Again this syllogism is a tautology for all occupancies of *a*, *b,* and *c* with their true and false values. If the following sentences for *a*, *b,* and *c* are taken:

a := *the gross national product increases.*
b := *the headcount steadily decreases.*
c := *the unemployment figure decreases.*

we get the following result for the conjunctive syllogism:

IF the gross national product increases, THEN the unemployment figure decreases.	$(a \rightarrow c)$	
conclusion: IF the gross national product increases AND the headcount steadily decreases THEN the unemployment figure decreases.	$(a \wedge b) \rightarrow c$	_12)

Although from a syntactical point of view both inference figures are tautologies and although in both cases the premises are correct (logically true) the conclusions in both examples are not correct.

The problem is given – from a logical point of view – by the differences between the so-called object-language (syntax) of the formulas on one side and the meta-language (semantics) on the other side, i.e., the meaning of the sentences attributed to the logical variables. These and a series of analogous cases confront every software engineer with nearly unsolvable problems.

Within the positive-linguistic framework of language a hierarchical order relation is presupposed which does not allow any mediation between the object- and the meta-languages. The term 'meta' already articulates that there is no co-ordination of languages. Co-ordination, i.e., heterachical structures are still not yet the focus of research in modern logic. Even the conceptions of so-called paraconsistent logics are exclusively positive-linguistic calculi. The problem of mediation, the problem of modeling, and the implementation of heterarchical process structures can never be solved on such a conceptual basis. [[51]]

A similar fundamental logic problem as given by the relations (11) and (12) is the well-known case of *ex falso sequitur quodlibet*, which says 'from falsity, concludes anything' (cf., table 12 – column 9, row 5 [B5]):

$$\sim a \rightarrow (a \rightarrow b)$$ [[52]].

If a is false and consequently $\sim a$ is true, it follows that a implies anything. This sentence which is (logically) true within the classical standard logic, is rejected by the logic of relevance[[53]] and in most paraconsistent types of logic.[[54]] However, the problem which

[51] See for example:
a) G. Priest, R. Routley, J. Norman (eds.) in: *Paraconsistent Logic – Essays on the Inconsistent*, Philosophia Verlag, München, 1989.
b) G. Priest, *In Contradiction – A Study of the Transconsistent*, Martinus Nijhoff Publ., Dodrecht, 1987.
c) Rudolf Kaehr, *Neue Tendenzen in der KI-Forschung - Metakritische Untersuchungen über den Stellenwert der Logik in der neueren Künstlichen-Intelligenz-Forschung*, Stiftung Warentest, 1980; in: www.vordenker.de
Note: The *Calculus of Indication* (CI) of Spencer Brown does not contribute anything to a solution of these logical problems. From a technical point of view, the CI can be considered as a non-autonomous two-valued calculus, which if implemented turns towards a conception that corresponds to Maxwell's demon which is very well-known from thermodynamics. On such a conceptual basis it is neither possible to construct a thermal engine nor will it be possible to build any computer (see also **[C5]**).

[52] The tilde character stands – as it was already mentioned above – for the classical negation, i.e., $\sim a$ or non-a.

[53] see for example: http://plato.stanford.edu/entries/logic-relevance/

[54] see for example: http://plato.stanford.edu/entries/logic-paraconsistent/

results from the meaning, i.e., the context dependency of the sentences attributed to the logical variables cannot be solved simply by rejecting the problem; it is only deferred.[[55]]

A satisfactory solution of these problems is only possible on the basis of mediating the object- and meta-language. However, this requires heterarchical structures of thought and/or the suitable formal logical tools. Günther's logic of place-values offers a first glance of how this problem can be attacked in the future. For implementation, however, the logic of place-values is not adequate, i.e., for any implementation the potential offered by kenogrammatics and morphogrammatics as well as by the theory of qualitative numbers has to be considered. But this would lead us too far astray within the context of the present discussion and therefore must be postponed for another occasion.

Summary

The introduction of heterarchy as a complementary category of description to the well-known and still unilaterally used term hierarchy – as soon as the complementarity of the two terms is recognized – leads to a much sharper scientific definition of both terms. A quite similar development could be observed in physics in the nineteen-thirties in the historical transition from Newtonian mechanics to quantum mechanics.

The fact that the concept of heterarchically structured processes has been ignored to the greatest extent possible by the mainstream of the scientific community may be explained by the far reaching scientific, philosophical, social, and technical consequences implied by the modeling and implementation of McCulloch's 'heterarchy of values' – the interplay of heterarchical and hierarchical processualities. Most notably, the insight that there are processes which cannot be represented on a sequential linear time axis and which cannot be described using the conventional methods of mathematics and logic. It is not possible to measure the interplay of heterarchically and hierarchically structured processes in the way that one can measure, for example, the intensity of electromagnetic waves. These processes elude any positive-linguistic framework of scientific language. Since all mental processes belong to this category of processuality which is obviously subject to living systems one can conclude that life in general is characterized by this type of processuality. Because of the dominance of natural sciences the belief that only measurable things are close to reality has been engrained deeply in our scientific thinking (especially in Western culture), so that imagining such a processuality still emerges as an intellectual problem.

Thus it is not surprising that the models and simulations of "intelligent" systems as they are offered today by artificial intelligence research are based exclusively on the Church-Turing-thesis and result in hierarchical process structures. In *Brain and Behavioral Sciences (BBS)* an intelligent and provocative discussion of his book *The Emperor's New Mind* – which was published in 1990 – with the *crème de la crème* of the AI research camp, Roger Penrose describes the situation as follows: [[56]]

> "It is a remarkable fact that any computational process whatever (that operates with finite discrete quantities) can be described as the action of some Turing machine. This, at least, is the contention of the so-called Church-Turing thesis, in its original

[55] see ref. 51c.

[56] R. Penrose, *Précis of The Emperor's New Mind: Concerning computers, minds, the laws of physics*, Behavioral and Brain Sciences (1990)13, 643-705.

> mathematical form. Support for this thesis comes partly from Turing's careful analysis of the kinds of operation one would actually consider as constituting a computational or algorithmic process, and partly from the striking fact that all the various alternative proposals for what an "algorithm" should mean (put forward at around the same time by Church, Kleene, Gödel, Post, and others) have turned out to be completely equivalent to one another. Some of these proposals had the initial appearance of being completely different, so their equivalence is a strong indication of the fact that they are merely alternative ways of describing an absolute abstract mathematical concept, that of computability, (which is independent of any particular realization of it that one may care to adopt.) In addition to an extended and detailed description of Turing machines, I give a brief description of Church's remarkable calculus in Emperor, pp. 66-70)."

Today it can be seen that completely new forms of thinking are required[[57]], if the complexity of such highly interwoven process structures as occur not only in our brains but also within our societies are to be modeled and implemented.

"*Unsere signifikanten Probleme können wir nicht auf der gleichen Ebene des Denkens lösen, auf der wir sie geschaffen haben.*"[[58]]

This citation accredited to Albert Einstein is more than actual today, however a change of our scientific paradigm has not yet been observed.

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[57] R. Kaehr, *Diskontexturalitäten: Wozu neue Formen des Denkens? Zur Kritik der logischen Voraussetzungen der Second Order Cybernetics und der Systemtheorie*.
in: http://www.vordenker.de/ggphilosophy/diskontext.htm

[58] Translated: " Our most significant problems cannot be solved at the same level of thinking at which we produced them".

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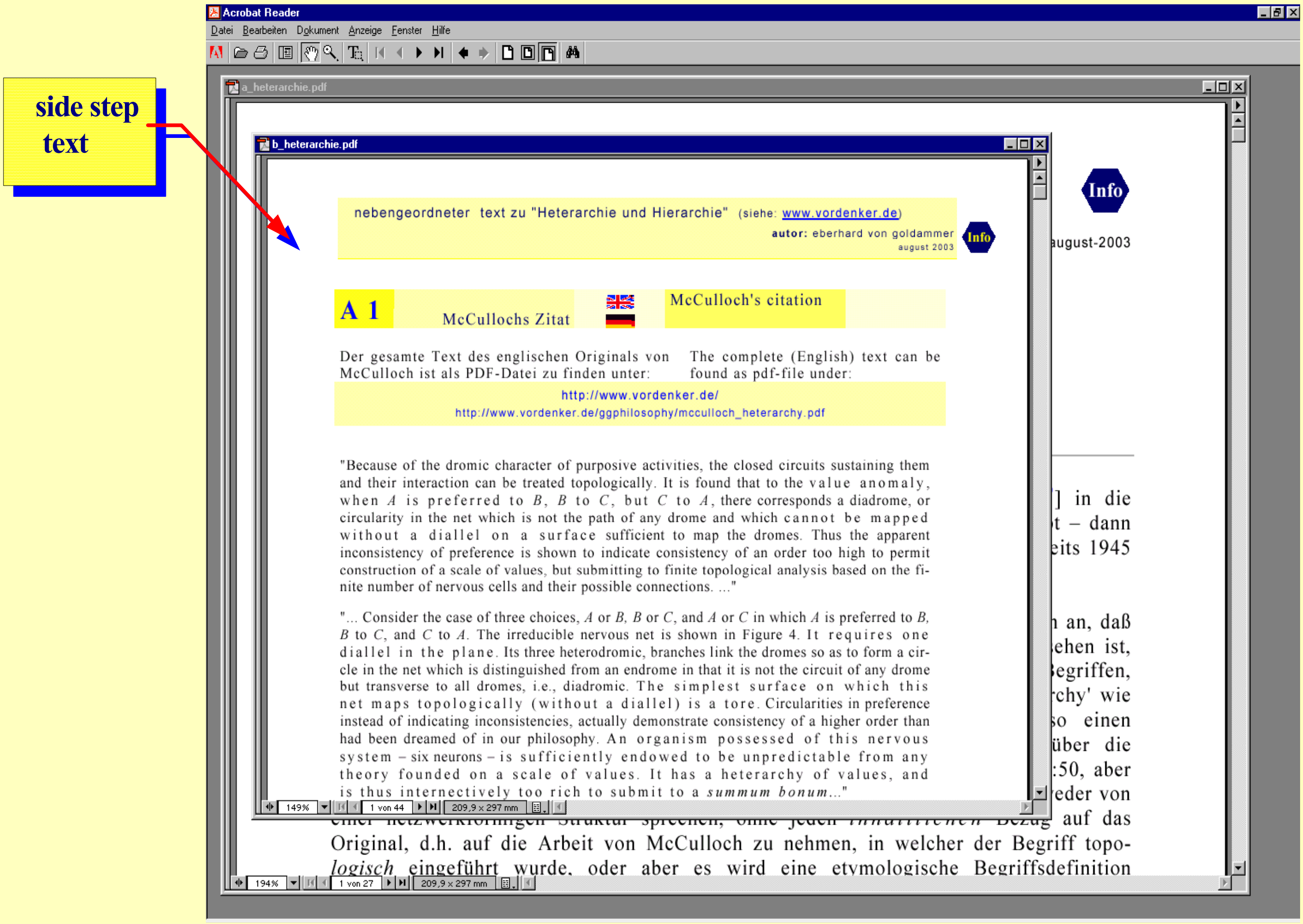

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