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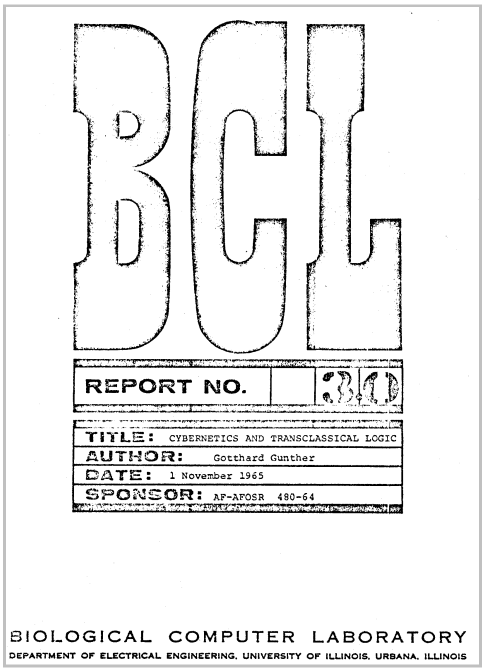
Cybernetics and the Transition from Classical to Trans-Classical Logic



The classic tradition of logic assumes that a total disjunction exists between a thought-process and its content, or between the observer and the observable: In order to arrive at an adequate description of the observables, scientific discourse has first to be drained of all subjective admixtures. Classic, two-valued logic implements the requirements of this disjunction perfectly. It provides a system where one value is considered to be positive and the other negative. In classic theory this distinction happens to coincide with the semantic distinction between designation and non-designation. We intend to show that this coincidence cannot be maintained in trans-classic systems of logic formed by the introduction of more than two values. It will be further shown that the separation of the dichotomies between affirmation and negation on one hand and of designation and non-designation on the other hand represents the very criterion by which the classical and the trans-classical theory of thinking may be distinguished. Finally, it will be noted that the distinction between the negational and the designational function of a value is of basic significance for the self-referential theory of computers.

We shall start with some remarks about the value aspect, and draw attention to the fact that classic logic produces an isomorphism between the set of all affirmative statements and the set of all negative statements. This isomorphism is based on the formal symmetry between affirmation and negation and the well-known principle of duality of two-valued logic. This permits the seemingly paradox statement that no semantic distinction exists between the positive statement and its negation.[1] Let us examine the famous classical example of:

Socrates is mortal
Socrates is not mortal.

It would, of course, be absurd to assume that there is no difference between the two propositions if we take their (contingent) material content into account. But the structural analysis of logic is not concerned with this content. It is only relevant that both propositions can be made to fit the fact by using exactly the

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1 Cf. Reinhold Baer: Hegel und die Mathematik. Verhandlungen des zweiten Hegel-Kongresses, Oct. 18-21; p. 1048 Tübingen (1932)

same negational operator transforming the affirmative proposition into the negative and the negative into the affirmative. This is due to the symmetry displayed by the classic table of negation.[2]

	N
1	2
2	1

Table I

Assume we have a many-valued logic with m values i (i = 1, 2, 3, ..., m) which are generated by a successor operation i' = i + 1 ($1 \leq i \leq m$) where the successor of m shall be, by definition, the initial value of the system, say, m' = 1.

What would be a proper negation in such a system? Starting from the classic exchange operation in Table I, it suggests itself to extend this *modus operandi* also into many-valued systems. Closer inspection, however, shows that in any m-valued logic only m – 1 independent negations $\mathbf{N}_i$ exist, namely those that produce an exchange operation with their immediate successors:

x	$\mathbf{N}_i$ (x)
1	2
2	1
⋮	⋮
i-1	i-1
i	i+1
i+1	i
i+2	i+2
⋮	⋮
m	m

Table II

Or, to put it differently:

$$\mathbf{N}_i(j) = \begin{cases} j & \text{if } j \neq i, \text{ or } j \neq i' \\ i & \text{if } j = i' \\ j' & \text{if } j = i \end{cases}$$

Table II may give the Impression that there are m independent negations because

$$\mathbf{N}_m(m) = 1$$

It can be shown, however, that $\mathbf{N}_m$ is not an independent operation because it can be produced by a combination of the preceding negators. We define independent negations as procedures obtaining successive values which have not previously occurred in the system. Only the *reversal* of an independent negation may obtain a value already extant in the system. But if $\mathbf{N}_m$(m), then *both* the negation and its reversal produce values which have occurred in

[2] Since we are going to introduce more than 2 values we follow the accepted usage in treatises of many-valued systems by using the positive integers 1, ..., m as Symbols for logical values. This means that 1 indicates what is considered the positive value in classic logic and 2 stands for its negation.

exchange operations $\mathbf{N}_{m-n}(m-n)$ where n < m. However, the denial of $\mathbf{N}_m(m)$ as independent negation gives us an opportunity to say what we mean if we distinguish between positive and negative values: a value is considered positive if it cannot be generated by an independent successor operation. Values that are generated by such an operation are considered to be negative. It follows that n-valued systems can only have one positive value and that all additional values must be considered negative. Thus, we arrive—in contrast to the classical system—at a basic asymmetry of affirmation and negation in many-valued logics.

We now turn our attention to the problem of designation. Since classic logic has only two values it is obvious that the dichotomy between designation and non-designation must, as we noted before, coincide with the distinction between positive and negative values. It will also be useful to remind ourselves that in classic logic a positive proposition is structurally identical with its own negation, because structural *differences* can only be generated by a *difference in the number of values* employed for a given purpose. The negative value of classic logic is, designationally speaking, only a "repeater-value." It merely iterates what is already available through the positive value. It follows that if we intend to designate something apart from what is already designated in classic logic we would have to use a plurality of values (at least two) for such an additional designational intent. And non-designation would have to repeat such a plurality. If we take into account the asymmetry between positive and negative values in many-valued systems it may be anticipated that the simple isomorphism that exists in the classic system between positive and negative values on the one hand, and designative and non-designative values on the other hand, does not hold any longer.

What would then be a proper definition of designation and non-designation in many-valued systems? Starting from classic logic where one set of values (in this special case, of course, only one) is deemed non-designative when repeated, it is in many-valued systems the *excess* of values after collecting complete designational systems of ascending valuedness that will be deemed non-designative. For it is precisely this excess of values that must repeat at least one of the previous collected (designational) systems.

We present in Table III as an example the designational value distribution of an m-valued system where m = 17.

Table III [3]

m	collected designational systems	excess of values
17	1 2 3 4 5	2

repetition

In such a logic five ascending modes of designation are available, starting with the one-valued designation of classic logic. This leaves an excess of two values

3 The reader is reminded that according to present concepts of meta-theory a single logical system cannot designate at the same time two objects of different structural types. It follows that, if a given ontology presents objects of different types available for potential resignation, *one* type must be singled out for the actual designational operation.

for non-designation. In other words: A logic with 17 values represents the special case where designation by two values is repeated in a two-valued non-designational system. Generalization of this procedure is easily obtained by noting that the number of non-designating values in an m-valued system is simply the difference between the sum of consecutive integers $1 \rightarrow k$:

$$\sum_{1}^{k} i = \frac{1}{2} k(k+1)$$

where $$k = \begin{cases} p = \frac{1}{2}\left(\sqrt{1+8m} - 1\right) & \text{if } p \text{ is integer} \\ \text{next integer} > p, \text{ if } p \text{ is not integer} \end{cases}$$

and m = the number of values available.

For convenience, the following Table IV, given for the first 10 values of m shows the number of positive and negative values (**N**$_+$, **N**$_-$) as well as the number of designative and non-designative values (**N**$_D$, **N**$_{ND}$).

Table IV

m	Negation		Designation	
	N_+	N_-	N_D	N_{ND}
1	1	0	1	0
2	1	1	1	1
3	1	2	3	0
4	1	3	3	1
5	1	4	3	2
6	1	5	6	0
7	1	6	6	1
8	1	7	6	2
9	1	8	6	3
10	1	9	10	0

Since in classic logic only one value is available for designation, all objects must—as far as their logical structure is concerned—belong to the same ontological category (one-valued ontology). But if we assume that the universe contains at least two basic categories of potential logical objects, namely systems without self-reference and systems with self-reference, two-valued logic has no means to distinguish them by designation. It is safe to assume that an object with the capacity of self-reference displays a higher structural complexity than one without this capacity. But in order to designate a difference in the complexity of ontological structure, the designational process itself has to show a corresponding difference. This, however, can only be effected—as we noted before—by a difference in the number of values employed for designation. On the other hand, we have shown that the number of positive valued in any m-valued system always remains 1. *It follows that the coincidence of the distinction between assertion and negation with the distinction between designation and non-designation can only hold in a two-valued system, of logic*. If we proceed to systems where m > 2, two cases may occur: either there is no excess of non-designational values or there is such an excess. An m-valued system which does not show an excess of non-designational values cannot be interpreted as a logic. It must be considered an ontology followed by logics which refer to it. The logics, of course, represent all those cases where trans-classical systems show a distinction between designative and non-designative values.

This is the point where we must establish the connection between trans-classic logic and cybernetics. Cybernetics is basically the theory of self-referential systems. On the other hand, it has been recognized at least since the time of Kant's *Critique of Pure Reason* that self-reference cannot properly be dealt with by two-valued traditional logic. We shall give only one basic reason: a two-valued calculus is unable to furnish a criterion for the distinction between *information* and *meaning*. Differences in information are structurally equivalent to the distinction between positive and negative values. Differences in meaning, on the other hand, are related to the distinction between designation and non-designation. To put it differently: *the non-coincidence of the negational and the designational function of values is formally equivalent to the difference between information and meaning*. Where both coincide, as in two-valued logic, structural characteristics interpretable as meaning are unavailable. If a computer produces a map and the map represents only information, two-valued logic is all that is required. This is beyond dispute. Information theory therefore states quite rightly that the meaningful aspect of the informational input into a system may be ignored by its computations. We only have to acknowledge the limitation that a system incapable of computing the difference between information and meaning can never possess self-reference.

In self-referential systems a map serves a double function: a) relative to the environment, and b) relative to the self-referential organization of the system. In case a) the relation is purely informational; in case b) it is hermeneutical. The relation of the map to the mappable object (ignoring the "subject" for which it *is* a map) is fully expressible in terms of two-valued logic. Self-reference, however, requires an "outside" observer who does not identify himself with either the map or the object, but is capable of comparing them. The concept of the object at which the map points belongs to the traditional one-valued classic ontology and requires therefore only a single value for designational purposes. The self-referential function of the observer, however, requires two distinctions: one between himself and map-and-object; and second between the map on one side and the object on the other side. The functional role of the observer as that which is excluded from the domain of the observables is represented by a non-designational value. Designated are only map and object. This requires, in order to keep object and map apart, and to indicate that the map *means* the object, two values for designation. But designation by more than one value is (as we know) only available in trans-classic many-valued systems of logic. The m-valued non-coincidence of informational and hermeneutic structure for all cases where $m > 2$ is a necessary but not sufficient prerequisite of any cybernetic theory of self-referential systems. What would be sufficient? To use an expression of Rudyard Kipling: "This is another story."

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