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Panel Discussion

TIME AND MEMORY

HEINZ VON FOERSTER *(University of Illinois, Urbana, Ill.):* I am sorry that Dr. Whitrow is absent this morning, because I hoped he would elaborate upon some of his delightful remarks of yesterday evening on the relationship between time and memory. Since I believe that this relationship may be of interest to some participants in this conference, I wish to add just a few words.

Dr. Whitrow said yesterday that we know little about "memory." I whole-heartly agree but I would like to add that we know even less about "time". The cause for this deficiency I see in the superior survival value for all perceptive and cognitive living organisms if they succeed to eliminate quickly all temporal aspects in a sequence of events or, in other words, if "time" is abandoned as early as possible in the chain of cognitive processes. I believe that I can give at least two plausible arguments to support this proposition. The first argument is purely numerical and attempts to show the infinitely superior economy of a "time-less" memory compared to a record which is isomorphic to the temporal flow of events. Consider a finite "universe" which may assume at any particular instant precisely one of n possible states, S_1; S_2; S_3; . . . S_n. Let m be the length of a sequence of states:

	1	2	3	4	....	m
e.g.:	S_{15}	S_{23}	S_4	S_{105}	....	S_{22}

The number, N, of distinguishable sequences of length m is equivalent to the number of combinations of n distinct objects taken m at a time, multiplied with the number of permutation of m distinct objects. This is because the sequence S_a S_b is, of course, different from the sequence S_b S_a. Hence

$$N = \binom{n}{m} \cdot m! = \frac{n!}{(n-m)!}$$

If both n and m are large numbers, but n is much larger than m, one may approximate

$$N \cong n^m$$

The number of binary relays to hold this number - or the "amount of information" to be stored - is approximately

$$H = \log_2 N = m \log_2 n = m H_0 \text{ bits},$$

where H_0 is the amount of information necessary to specify one state of the universe. As an example, let us consider the retinal mosaic of various excited states of its rods and cones as the states of our "visual universe". Conservative estimates suggest 32 distinguishable states for each receptor of which there are about two hundred millions in both eyes. Hence, the visual universe has

$$n = (32)^{2 \cdot 10^8}$$

distinguishable states and

$$H_0 = \log_2 n = 10^9 \text{ bits}$$

If only ten states per second are processed by the retina - a regular movie projector presents 24 pictures per second - then during one second a sequence of length m = 10 is to be stored, i.e.,

$$H\ (1\ \text{second}) = 10^{10}\ \text{bits}$$

However, the entire brain has "only" 10^{10} neurons at its disposal. Let us be optimistic and assume one thousand bits stored within each neuron, then in one thousand seconds - or slightly over a quarter of an hour - the whole brain is flooded with information, most of which may be completely worthless.

On the other hand, if it were possible to integrate the sequence of a events into a single "operator" that permits reconstruction of the sequence, say, "a galloping elephant," "a flight over the Atlantic," etc., the number of distinguishable "macro-states" becomes

$$N^* = \frac{n}{m}$$

and

$$H^* = H_0 - \log_2 m$$

The compression ratio between these two methods of "storage" is simply

$$\frac{H^*}{H} = \frac{1}{m}\left(1 - \frac{\log_2 m}{\log_2 n}\right) \cong \frac{1}{m}$$

If the length of the sequence is extended, this reduction in uncertainty may take on values of considerable magnitude.

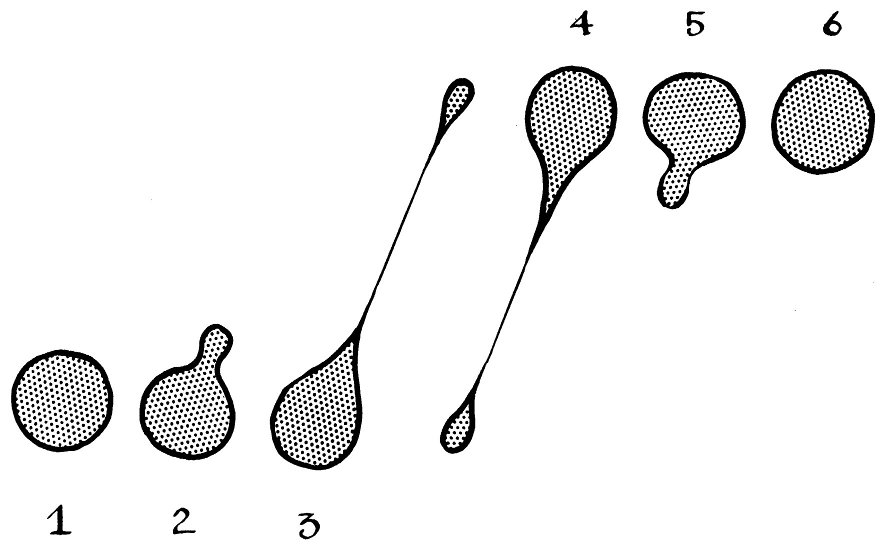

FIGURE_1. Schematic representation of a unicellular animal moving from one spot to another by extending a tubular pseudopod and pulling itself up through this extended capillary.

I hasten to demystify these "operators" which I have just mentioned. Indeed, they are perpetually computed in our perceptive system and their abstracting powers become apparent in their linguistic representation, usually in form of *names* for spatial abstracts and of *verbs* for temporal abstracts. In fact, without these abstracting operators we could not conceive of motion or of change, and - as an ultimate abstraction - of the flow of time. Contemplate for a moment the somewhat formalized representation of a unicellular animal moving from one spot to another by extending a tubular pseudopod and pulling itself up through this extended capillary (See FIGURE 1, Stages 1-6). However, what we see in fact is a sequence of six apparitions of quite distinct shapes of which it is difficult-I believe even impossible-to assert that they represent the "same object" unless, of course, the set of transformations is specified under which the properties of this "object" remain invariant. These transformations may accommodate spatial as well as temporal aspects of the object under consideration, as can be seen by the linguistic representation of the totality of events depicted in FIGURE 1: "a unicellular animal moving from one spot to another by extending a tubular pseudopod and pulling itself up through this extended capillary." The invariance of the sequence 1, 2, 3, 4, 5, 6 is suggested by a feeling of "inappropriateness" if any permutation of this sequence, say, 2 1 4 3 5 6 is proposed as an alternate possibility.

It is clear that it is due to memory that temporal abstracts can be computed and stored. Although memories may have some charming aspects, their crucial test lies in their efficacy to anticipate sequences of events, in other words, to permit inductive inferences. The conceptual construct of "time" is, so far as I see it, just a by-product of our memory, which in some instances may use "time" as a convenient parameter - a *tertium comparatum,* so to say - to indicate synchronism of events belonging to two or more spatially separated sequences. Of course, there is *no need* to refer to time in such comparison, for it is always sufficient to take one sequence as "standard" and to associate with standard events the events of another sequence as, for instance, the anticipation of the sequence of events regarding Peter:[1] "...Verily I say unto thee, that this day, even in this night, before the cock crows twice, thou shalt deny me thrice."

In parenthesis it may be interesting to note that this prediction would immediately lose its punch if it would use temporal reference, for instance: "... even in this night, before 6:30 a.m., thou shalt deny me thrice." The intellectual slump one suffers in this version stems, I believe, from the fact that idealized absolute, or Newtonian, time carries no information: $H \rightarrow 0$. Unfortunately, in spite of considerable efforts by clock designers to build a perfect clock, this ideal goal has not yet been attained. There is still a small residue of uncertainty $H = \varepsilon > 0$, which is due to small inaccuracies of even the best clocks and, ultimately, there will be quantum noise which will set an absolute limit to this enterprise.

Since these assertions which represent my second argument may, at first glance, sound surprising, let me first demonstrate that Newtonian time is a useful but unnecessary parameter in a complete description of the universe; second, let me briefly state what I mean by an accurate clock.

In practice an approximation to Newtonian absolute time, called "ephemeris time," is obtained in two steps as Dr. McVittie pointed out earlier this morning. First the equations of motion in Newtonian mechanics are solved for various celestial bodies, in particular for the different planets P_i. These solutions usually express the positions r_i of these bodies in terms of a linear parameter, t, called time:

$$\vec{r}_i = \vec{r}_i(t)$$

Second, the scale of this parameter is adjusted so as to give the best fit between observation and the theoretical solutions. In fact, this established scale fits the observations so well that there is a residual error of only one unit in about 10^{11} units.[2] With this estimate one may calculate the uncertainty H of reading this astronomical clock. With the probability p of making an error

$$p = 10^{-11}$$

and with the definition of H for a binary choice:

$$H = - p \log_2 p - (1 - p) \log_2 (1 - p),$$

one finds the uncertainty to be

$$H = 3.8\ 10^{-10} \text{ bits/ unit.}$$

It may be interesting to determine whether this small residue of uncertainty is due to "noise" in the observed system, that is the planets refuse to obey Newton by occasionally performing small extravagancies in their otherwise predictable behavior, or whether this noise is introduced by the transmission channel, that is by inaccuracies of observation due to random fluctuations in the atmosphere, in the optical equipment or in the evaluation of data.

If the latter should be the case, then this uncertainty can be made arbitrarily small on the basis of Shannon's[3] far reaching theorem which states that if the same sequence of signals is repeated again and again the effect of errors (within certain limits) that are introduced during transmission can be made arbitrarily small. This principle of "Reliability through Redundancy" is most effectively applied in all periodic or repetitive phenomena. All "clocks," for instance, are based on periodic event sequences, and environmental periodicities can be recognized, or absorbed, by the genetic code of reproducing and mutating living organism by programming these periodicities into the organism in the form of circadian rhythms: "Even if you read me poorly, if you read me often enough, you will get my message."

If I should make a guess as to the causes of the small residue of uncertainty left in establishing ephemeris time, I would venture to say that they come indeed from a certain capriciousness of the planets, for the channel noise has most probably been eliminated by the prolonged observation of these periodic events.

After having assured ourselves that this parameter "time" is universal*) and does not change scale from planet to planet, we can now eliminate it from the set of equations which represent the positions of the planets as a function of this parameter by selecting the positions of one planet $\vec{r}_0$ as reference for the positions of all others:

$$\vec{r}_i = R_i(\vec{r}_0)$$

In other words, one would, for instance, tell the position of Mars in reference to the position of Venus, etc. Adhering to this scheme, appointments would be made in this form: "... we shall meet after the sun has risen twice." Of course, this is precisely the method outlined earlier in which simultaneity of events belonging to different sequences are used to establish the "when" of an event of one sequence by reference to a particular event of a standard sequence.

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*) That is within the framework of Newton's equations of motion.

For practical purposes, however, it is convenient to have a highly redundant signal generator - a reliable clock - which facilitates the estimates of the simultaneity of events in a large number of sequences. Clearly, such a device should not inject into the universe of observation unwanted uncertainties, i.e., each subsequent state should be well determined by its predecessor. This is most easily accomplished if this device goes at a constant rate, which gives the additional advantage that such a device may read ephemeris time which is a useful parameter in Newton's equations of motion.

This brings me to my second point, namely, what do I mean when I speak of a "reliable," of an "accurate," clock that goes at a "uniform rate". As Dr. McVittie has already pointed out, comparison of one clock with another may never establish which one goes ahead and which one falls back. Since cross-correlation between two clocks leads us nowhere, I propose to consider for a moment auto-correlation of one clock with itself.

FIGURE 2 shows such an arrangement where an illuminated diagonal slit in a circular rotating disk, representing the clock, is located at the center of curvature R of a convex mirror. In this arrangement, the clock is optically mapped onto itself.[+)]

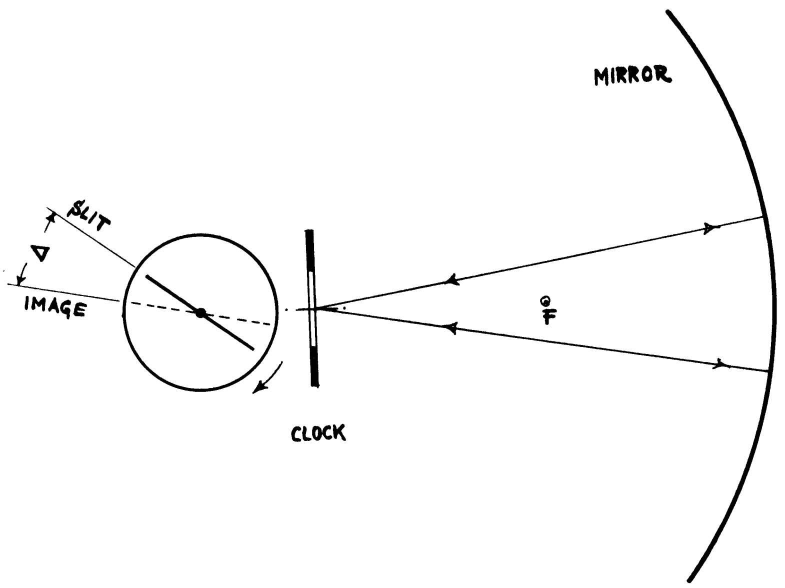

FIGURE 2. Determination of the accuracy of a clock through auto-correlation.

If the disc rotates with angular velocity ω, the angular position ϕ_1 of the slit and ϕ_2 of its image are given in parametric form (time t is parameter):

$$\phi_1 = \omega \cdot t$$
$$\phi_2 = \omega(t - 2R/c)$$

where c is the velocity of light. Eliminating t from this pair of equations expresses the position of the slit's image as a function of the position of the slit:

$$\phi_2 = \phi_1 - 2R\omega/c$$

[+)] The inversion is compensated by making the hand a diagonal slit rather than a radial one.

This means that the slit will be trailed by its image at an angle of

$$\phi_2 - \phi_1 \equiv \Delta = - 2R\omega/c$$

If the curvature of the mirror, its distance from the slit and the velocity of light are for the moment assumed to be constant, then it appears that the angular displacement Δ between slit and image will reflect all variations in the rate by which this clock proceeds. If ω increases, so will Δ and conversely, a reduced w will result in a smaller displacement. One is tempted to say that a constant Δ indeed ascertains an accurate clock that proceeds at a constant rate.

Nevertheless, this naive interpretation has been subjected to a severe criticism by E. A. Milne[++)] who rightly points out that a constant displacement Δ means only that its variation vanishes. With

$$\Delta = - 2R\omega/c$$

this means that

$$\delta\Delta = 0$$

or

$$\frac{\delta\Delta}{\delta R} dR + \frac{\delta\Delta}{\delta\omega} d\omega + \frac{\delta\Delta}{\delta c} dc = 0$$

Calculating the partial derivatives suggested above, this expression may be rewritten to read

$$\frac{d\omega}{\omega} + \frac{dR}{R} = \frac{dc}{c}$$

if in spite of constancy of the displacement a dependency of ω with the position ϕ_1 of the slot is suspected, one may write:

$$\frac{1}{\omega}\frac{d\omega}{d\phi_1} + \frac{1}{R}\frac{dR}{d\phi_1} = \frac{1}{c}\frac{dc}{d\phi_1}$$

This expression suggests that indeed a variation of rate at which the clock proceeds

$$\frac{d\omega}{d\phi_1} \neq 0$$

may go unnoticed by relying on an invariable displacement Δ, if the mirror flaps or wiggles, or if the velocity of light jerks back and forth precisely in such a manner as to compensate for the variation of ω. One may imagine a "law of nature" that couples the three quantities R, c and ϕ_1 so that the relation obtains

$$\frac{1}{c}\frac{dc}{d\phi_1} - \frac{1}{R}\frac{dR}{d\phi_1} = \Omega(\phi_1)$$

where the function on the right hand side represents the expression

$$\Omega(\phi_1) = \frac{1}{\omega(\phi_1)} \cdot \frac{d\omega(\phi_1)}{d\phi_1)}$$

[++)] Milne's criticism is addressed against a "Gedanken experiment" which incorporates entirely different physical devices. However, the basic features in both experiments are equivalent.

There are, of course, an infinite number of functions $c(\phi_1)$ or $R(\phi_1)$ or couplings between R and c, which will satisfy the above differential equation.

However, the amusing upshot of this side issue is that whatever these laws may be, they must transmit with great accuracy the fluctuations in the clock $\omega(\phi_1)$ to the mirror and force it to wiggle or to flap, or to the velocity of light to jiggle in precise compensation of the fluctuation of the clock so as to make the deviation Δ to appear unchanged or, at least, to change so little that ephemeris time can still be determined with the great precision mentioned earlier. Hence, these processes - imaginary or real - are of such redundancy that they do not interfere with our use of the parameter "time" as a *tertium comparatum* which facilitates highly accurate determinations of the simultaneity of events belonging to different cognitive sequences.

References

1. ST. MARK: 14-30.
2. G. M. McVITTIE. 1961. Fact and Theory in Cosmology : 25. Eyre & Spottiswoode. London. (1961).
3. C. E. SHANNON & W. WEAVER. 1949. The Mathematical Theory of Communication. : 39. University of Illinois Press. Urbana, *Ill*.