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Gotthard Günther [*]

THE LOGICAL PARALLAX

I.

A most exasperating situation has developed in modern logic during the last two or three decades. On the one hand it has been pointed out with absolutely indisputable arguments that the only possible logic of Man is Aristotelian, that is two-valued. On the other hand a considerable number of many-valued calculi of symbolic logic have been developed during the same period, and their proponents take it as a matter of course that they have discovered a new non-Aristotelian system of rational thinking.

Something must be wrong here. If the Aristotelian logic is the only system of thinking man can ever use, then it should not be possible at all to develop many-valued calculi of symbolic logic. But the existence of three-valued, four-valued and generally n-valued calculi is an undeniable fact. And nobody can contradict factual existence. So the camp followers of Aristotle *must* be wrong. However, the defenders of the two-valued system are not impressed at all by the foregoing argument. "Have you ever," they usually retort, "really attempted to think with, let us say, a seven-valued logic? Well, you can't, no matter how hard you try. It is an absolute psychological impossibility. And that is a fact, too."

Obviously we have reached an impasse. If both sides are able to appeal to the authority of existing facts, then only one conclusion is permissible: the general basis of the whole controversy is unacceptable; or, to put it more technically, both sides share a logical premise that is false. The whole controversy, of course, is based on the mutually agreed assumption that in the case "two-valued system versus many-valued system" different forms of logic compete with each other. It is tacitly assumed that the Aristotelian theory of thinking is opposed and may be superseded by a non-Aristotelian mentality of man which has finally come to the surface after several thousand years of human history.

However, this general premise that the coexistence of two- and many-valued calculi indicates a case of competing logics must be wrong, if it leads us to the predicament that both sides are able to retrench behind unassailable facts, and to take up positions from which they can never be dislodged. In order to get on with the problem we have, therefore, no other choice but to discard the general assumption that the whole discussion revolves around the alternative Aristotelian logic contra Null-A logic. So far so good.

But now a very different sort of trouble – slightly on the ridiculous side – turns up. We have no issue left at all! What on earth could be the difference between

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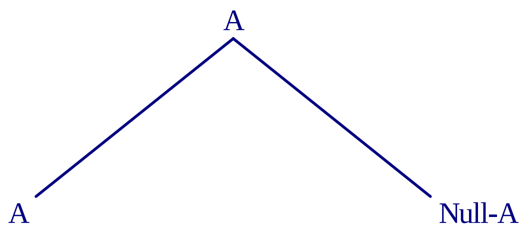
* aus: Astounding Science Fiction (Editor: John W. Campbell, jr.), vol. LII, number 2, November 1953, p.123-133.

Anmerkung im Original: Dr. Günther's work on multi-valued logic is now attracting acutely interested attention among logicians; this is a general discussion of what it means logically to say that a thing is both true and not-true.

a two-valued, and a many-valued symbolic calculus if not a difference of logic and rational meaning? It stands to reason there must be one. But it is equally clear that we have not discovered it yet. The only thing we can do now is to re-examine the positions of the two logical schools of thinking.

All Aristotelian claims essentially boil down to one very impressive argument. First, it is assumed that two different systems of logic, Aristotelian and non-Aristotelian, do coexist. But if this is the case, then we obtain immediately an Aristotelian alternative between the Aristotelian system A and the competing system Null-A. Because whenever a factual problem occurs we shall have to *decide* whether it has to be solved with the help of A or of Null-A. Our decision will either be true or false. In other words: it is again a two-valued logic which decides between A and Null-A. A simple diagram may illustrate this interesting property of our traditional logic:



In more technical language: an assumed plurality of logical systems leads only to a re-iteration of the Aristotelian logic. In our diagram the traditional two-valued logic A shows up twice. The lower occurrence indicates the theoretical level, the one on top the action-decision-level. This is significant because it shows an essential difference within the concept of A. At any rate it is impossible to get away from the fact that the Aristotelian system A contains itself and any other hypothetical logic as subsystems within itself.

But what about the claims of the many-valued calculi. Their case seems to be almost as strong as that of their opponent. The case of Null-A essentially rests on the following argument: If you formalize the Aristotelian logic, you get a two-valued calculus. Now there is no logical reason why anyone should stop at the number "2" when introducing logical values. From the viewpoint of the calculus the number "2" is as arbitrary as any other number in the natural system of numbers. We might as well demand from people that they stop counting when they have reached the number three hundred sixty-five, because it is the number of days in a year. But what about the man who has four hundred dollars in his purse? There is indeed no reason at all – neither on Earth nor in heaven – why symbolic calculi of logic should stop at the number of two values. "2" is just an arbitrary number and has in the system of symbolic calculi no more and no less right than "7" or "60,000." Many-valued calculi are here to stay and all orthodox Aristotelians had better get reconciled to that prospect.

But that does not permit the conclusion that a many-valued calculus of symbolic logic necessarily represents a new non-Aristotelian system of logic, a higher system of reasoning which the Man of the Future might finally grow into. The very origin of the many-valued calculi of logic suggests a different interpretation. How did symbolic calculi generally come into existence? Well, there was first the actual Aristotelian *practice* of thinking, and after man had employed that practice for several thousand years he finally discovered the technique of formalizing his processes of thinking, and of expressing them as – two-valued – symbolic calculus. This happened quite recently. In fact not before 1854! (Cf. George Boole, An Investigation of the Laws of Thought. London 1854). But did the many-valued calculi originate in the same way? Did man first develop new habits of actual thinking, and after he had done so, formalize them into new many-valued calculi of logic? Most certainly not! All so-called non-Aristotelian calculi were produced as formal generalizations of the two-valued system with *no reference to actual human thought processes.*

This difference of origin establishes an enormous semantical distinction between the two-valued system and all its many-valued amplifications and generalizations. The two-valued calculus represents an *interpreted* system. An interpreted system is one where we know exactly what its formulas mean in terms of factual existence. An uninterpreted system is one whose formulas are developed without any regard to the meaning they may carry. They may have an objective meaning or they may not. We do not know it. The interpretational contents of the two-valued symbolic logic are the actual thinking-events that occur in every individual consciousness. Aristotle was the 1st to observe these events that take place in every individual subject of rational behavior, and the subsequently developed two-valued calculus describes the exact laws these thinking-events are governed by. For this very reason the two-valued calculus is called an interpreted system. Its interpretation is the fact that it describes in formal symbolic terms the actual events of rational human thinking for any given individual.

However, the many-valued calculi were not developed in accordance with that semantical pattern. There is no actual thinking these calculi are meant to depict. They came into existence by a very abstract and formal generalization of the two valued system. It follows that all many-valued calculi are as yet *uninterpreted* symbolic patterns of *unknown facts!* But what do these systems describe? The two-valued calculus describes human thinking as it occurs in every rational consciousness. We shall, therefore, from now on insist, that "thinking" and "Aristotelian thinking "are exactly equivalent terms. *A many-valued calculus does not designate any form of actual thinking.* This much is absolutely certain. But what does it designate? The second part of this article shall give the answer to that question.

II.

Have you ever stopped to reflect how semantically ambiguous the term "the universe" is? If we speak about the universe we mean, of course, the all-comprehensive realm of existence in our space and time. In other words, the term "the universe" denotes the sum of all events and *all* objects that have been in existence, that are in existence, and that ever shall be in existence. So far there seems to be nothing ambiguous about this concept. But let us look at the semantical implication of the idea and the trouble will show up at once. I, the present writer, say: "the Universe." This concept contains besides everything else my own body – including my brain – but, strange to say, it does *not* include my consciousness – whatever that may be. It simply cannot do so, because it is *a content* of my consciousness.

The very fact that I conceive this term excludes my conception from it. To be sure, the consciousness of everybody else – living or dead or yet unborn – is included in my concept of the universe. But my own thinking, that produces the term, is unconditionally excluded. In philosophical language: my own thinking represents the *subject* that conceives the *object* "universe." That might be tolerated as a regrettable but unavoidable incompleteness of our term, were it not for the fact that I am not the only one who does his own thinking and who conceives the general conception of the universe.

Albert Einstein, for instance, does his own thinking! It stands to reason that *his* conception of the universe contains the present writer completely with all his thought processes, in short, with his whole subjectivity. The subject "Gotthard Günther" is fully included in the Einsteinian Universe. But not the subject "Albert Einstein"! As far as he produces the idea "the Universe" the Einsteinian thought process is excluded from that term. The term remains the same, but its definable contents vary according to the person who conceives it. We thus arrive at three different meanings for the term "the Universe."

Let "U" represent the meaning of the concept without any reference to any thought process, and "U^S" my idea of the universe, and "U^O" that of any other person – for instance, Einstein. You can then postulate the simple equations:

$$U = U^S \qquad (1)$$

$$U = U^O \qquad (2)$$

These equations mean that I claim my idea of the universe is correct, and the other person makes exactly the same claim. It follows that, if we use the equations 1 and 2 as premises of a syllogism, we should be entitled to the conclusion:

$$U^S = U^O \qquad (3)$$

But equation (3) is false. Instead, it holds:

$$U^S \neq U^O \qquad (4)$$

In words: "U^S" and "U^O" are not identical concepts. To use our former example again: "U^S" defines me, the present writer, as the subject and Mr. Einstein is wholly included in "U" whereas "U^O" defines Einstein as subject, and I am completely included – with all my thought processes – in " U".

The situation is not entirely new. It has its corollary in astronomy. If we speak offhandedly of the location of a star, three different meanings of the term "location" are implied. A, The objective location of the star with reference to the galactic system it belongs to; B, my visual location of it; and C, the visual location for a second observer at a different point in space. The difference between B and C is called the parallax of the star. All observers agree as to the hypothetical data of A. But A is not an observational fact. Its value can only be deduced from the observed data of B and C. This is important. Although we all potentially agree about the objective location of a star, A, all actual scientific experience accessible to us is represented by B and C.

Exactly the same is the case with our general logical reference to the universe as an objective totality. Here again we agree that the "absolute" term "U" should have exactly the same meaning for anybody. To return again to our former example: "U" should contain the mental processes of "Einstein" and "Günther" at the same time. But "U" is logically accessible to us only as "U^S" and "U^O". *We therefore have to conclude that the universe is given us only under the condition of a logical parallax.* Its contents vary to a certain degree if we shift from the logical observer "$\ldots^S$" to any second subjective viewpoint "$\ldots^O$".

This has amazing logical implications and throws a revealing light on the mysterious role of the many-valued calculi. We shall now demonstrate the case of the logical parallax with the help of a very simple Aristotelian concept, the logical term "and." The meaning of "and" is expressed in symbolic two-valued logic with the help of the truth-table

p	q	p&q
T	T	T
T	F	F
F	T	F
F	F	F

(I)

Let "T" mean "true" and "F" correspondingly "false," then table I tells us that "p" *and* "q" (p & q) are only true together, when "p" and "q" are singly true. Therefore, the case of a true conjunction of "p" and "q" occurs only in the first line of the truth columns. A practical example may demonstrate that. Let "p" mean: "Roosevelt is dead" and "q" may stand for "Stalin is dead"; then the compound statement " Roosevelt *and* Stalin are dead" (p & q) will only be true if the truth-value "T" attaches to "p" as well as "q." In all the other cases the compound-statement will be false. (For more details cf. the article "Symbolic Logic and Metamathematics," by Crispin Kim-Bradley, Astounding, XLVIII, 6 pp. 94-102.)

Up to now it has been tacitly assumed that if two different persons use the table I no shift of meaning occurs that might affect the functional characteristics of our table. But now let us look at the following conjunctional statement:

I and the universe exist. A

If the present writer pronounces it, it has the meaning:

Günther and the universe exist. B

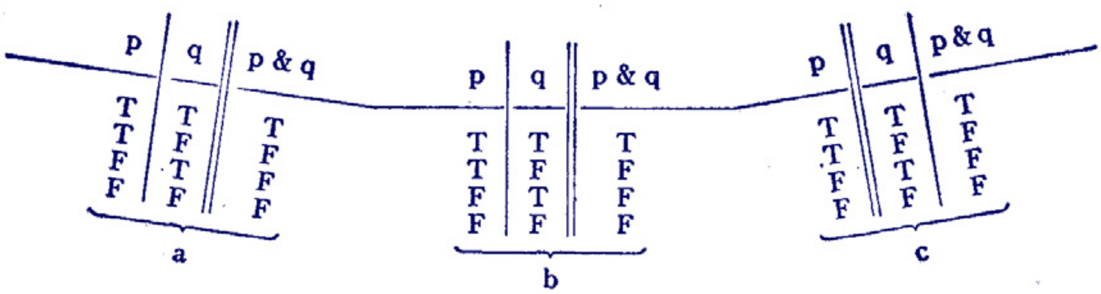
If Einstein makes the statement A, it necessarily means:

Einstein and the universe exist. C

This difference cannot be expressed by the truth-table (I). That is why the statement A has been regarded as meaningless within the context of symbolic logic up to now.

But let us now assume that relative to A the logical meaning of "and" in B and C is somehow "displaced."

We will illustrate that "displacement" in a naive, figurative way by writing down three identical truthtables and tilting them against each other:



This arrangement is intended to convey the idea that A the universe in general is Aristotelian; B my view of it is also Aristotelian; and C the view of any other person I might choose is again determined by Aristotelian categories. But these categories are slightly displaced relative to each other and produce different viewpoints.

It is, of course, impossible to express in geometrical angles the displacement a logical concept is bound to suffer if conceived by different individuals.

But this is the point where the theory of the many-valued calculi takes over ... and where we discover the meaning and the function of Null-A. In order to express the displacement our logical term "and" is subjected to if we shift from A to B or C we have merely to repeat the above truthfunction in a new table which contains a new, a third value between "T" and "F." We call it the displacement value and indicate it with the letter combination "Dspl." This leads us to a three-valued truth-function and we discover that the Aristotelian meaning of "and " unexpectedly appears in the shape of *three* different truth-functions:*)

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*) There is a very elementary way to construct table II. we number our values "T" = 1, "Dspl"=2, and "F"=3. We then discover that in table I the value "3" is the preferential one. It means "3" as the highest value is always chosen when available in the columns "p" and "q." However, as soon as we have three values, as in table II, six different preferential orders are possible. They are:

3	2	1
3	1	2
2	3	1
2	1	3
1	3	2
1	2	3

If the sum of the first two values of one of these preferential orders is bigger than the sum of the last two values, then the order belongs to the group of conjunctions. This is indeed the case for the first three preferential orders. The values for "$p\&^a q$" are then chosen by putting the value "F" in the truth-function column, whenever available in the "p" and "q" columns. If no "F" is available, then "Dspl"=2 is chosen, and "T" is only used if neither "F" nor "Dspl" is available. For the second function "$p\&^b q$" the value "F"=3 is again first choice. But, if not available, "T"=1 is chosen and so on.

p	q	p&^{a}q	p&^{b}q	p&^{c}q
T	T	T	T	T
T	Dspl	Dspl	T	Dspl
T	F	F	F	F
Dspl	T	Dspl	T	Dspl
Dspl	Dspl	Dspl	Dspl	Dspl
Dspl	F	F	F	Dspl
F	T	F	F	F
F	Dspl	F	F	Dspl
F	F	F	F	F

(II)

If one takes the pains to compare table II very carefully with table I, one cannot help noticing that whenever the "p" and "q" columns show exclusively. "T" and "F" values all three truth-functions of II agree completely with the two-valued meaning of "and." In other words: as far as the Aristotelian alternative of "true" and "false" is concerned our three-valued functions are identical with each other as well as with the traditional two-valued meaning of "and."

At this juncture the patient reader might very well ask: "if the three-valued table merely repeats our familiar Aristotelian meaning of "and," what is the use of that cumbersome table II? The point is well taken. Table II cannot be used to develop a logic of Null-A. As far as pure logical meaning is concerned it only repeats what we already know. However, it provides us with the principles of a displacement calculus by dint of which we can express in exact theoretical formulas the logical parallax inherent in the application of our Aristotelian logic. This is the significance of the three-valued and – as we shall see presently – of all many-valued calculi. The third and last part of this article shall demonstrate, how logical displacements of meaning are calculated, and shall moreover draw some general philosophical conclusions.

III.

In order to operate our traditional two-valued logic we need a so-called operator. This operator, usually called negation, transforms one value into its opposite. The Aristotelian negation is defined by the table:

p	~p
T	F
F	T

(III)

The table indicates that if "p" is true, then "~p" non-p is "false," and vice versa. It is obvious that you cannot calculate displacement values with the help of table III. In order to be able to do so we now replace our single Aristotelian negation by two half-negations which shall be defined by the two sub-tables III^1 and III^2:

p	~ p
T	Dspl
Dspl	T

(III[1])

p	~' p
Dspl	F
F	Dspl

(III[2])

That means: if I negate "p" by writing "~p" I do not obtain any more the value "false" provided "p" is true. From now on I obtain only the intermediate displacement value "Dspl". In order to arrive at "F" I shall have to complete a second half-step of negation "~'p".

The two tables III^1 and III^2 are all that is needed to calculate the logical parallax between the objective meaning A of "and," my personally restricted viewpoint B of it, and the viewpoint C of any third person. By using our half-negations we discover that the shift from A to B is expressed by the formula

$$p \,\&^{a}\, q \equiv \sim(\sim p \,\&\, {}^{b}\!\sim q) \qquad (5)$$

To calculate the subjective parallax between A and C we use the second half-negation and obtain

$$p \,\&^{a}\, q \equiv \sim'(\sim' p \,\&^{c} \sim' q) \qquad (6)\ ^{*})$$

The meaning of "and" remains exactly the same in all three cases of A, B, and C. It is impossible that it should be otherwise, because every time the "p" and "q" columns carry exclusively "T" and "F" values the result will be invariably the same as in the original table I. The formulas 5 and 6 measure only the degree of the displacement relative to some objective standard A. Thus formula 5 indicates that between A and B there exists a displacement of a half value, whereas A and B are a full value apart.

Thus the three-valued calculus of symbolic logic becomes an *interpreted* system. Its interpretation is not, that it reveals the structure of a new non-Aristotelian logic. It is no new logic but *a system of transformations* by dint of which different logical viewpoints can be calculated and translated into, each other. The three-valued calculus deals exclusively with the subjective differences between human beings as to their judgments of the surrounding world. What has been said with regard to the three-valued calculus applies – with proper generalization to any many-valued calculus of symbolic logic.

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*) These formulas are obtainable by the simple method of constructing tables with negated "p" and "q" instead of positive "p" and "q." The appropriate conjunction of table II is then executed and the resulting function is again negated with the same negational operator. The logical displacement between "$p\&^{b}q$" and "$p\&^{c}q$" involves a more complicated negational pattern. For those interested in the specific technique of a three-valued logic the two relevant formulas are herewith given:

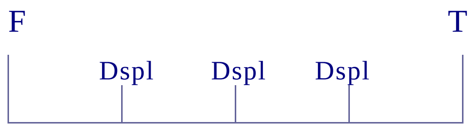
$$p\&^{b}q \equiv \sim(\sim' [\sim' (\sim p)\&^{c}\sim'(\sim q)])$$

$$p\&^{c}q \equiv \sim' (\sim[\sim (\sim' p)\&^{b}\sim(\sim' q)])$$

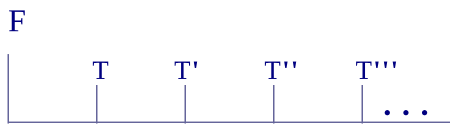
These formulas are more involved because the relations between two different individual viewpoints are always more complicated than the relation between the objective world A and one subjective viewpoint B or the other C.

There are cases when the displacement of rational principles is undoubtedly much larger than between different human viewpoints. For instance: between human and animal intelligence. It is certain, that in the latter case we should need a calculus with more than three values. And again a much higher number of values would be needed if we wanted to calculate the relative displacement of intelligence as subjectively expressed in the human consciousness and objectively displayed in the rational structure of crystals.

Let me conclude this article with some further perspectives. Table II is not the only possible model of a three-valued truth-table. A second and different type is feasible. Table II is to be used only if we deal with displacement problems of human or subhuman (animal, plant ...) intelligence. It handles only logical parallaxes expressible in *fractional* truth-values up to one full value. The following diagrams may demonstrate the essential difference:



and



The first diagram indicates that the whole range of one system is covered by the comprehensive alternative "true-false," and all displacement-values of many-valued calculi are located *inside* the simple F-T range. It stands to reason that the more "Dspl" values are introduced the smaller our negational fractions will become. (Incidentally: taking the rational human consciousness with the full F-T range as standard measure, a consciousness described by very small negational fractions will be so much dimmer!) However, it is possible to construct a three- or many-valued calculus where the third – and any additional – value is placed *outside* the range of F-T. The basic truth-tables of such systems, of course, vary greatly from our table II.

Let us call calculi based on this idea "systems of T-plurality." A first-order calculus of such a system would have the range F-T. The next would cover F-T', and so on. Viewed from the range of F-T" the values "T" and "T'" would then not appear as *displacement – but as reduction* values of "T". What is interesting about these systems of T-plurality is that they indicate a mentality higher than that of human individuals. There may be super-intelligent races in the universe which possess such higher spiritual faculties. However, this we do not know.

There is also the possibility that man himself may finally reach a stage of T-plurality intelligence. Yet for the time being, anyway, T-plurality calculi have no practical application, in contrast to displacement calculi, which can be used in their three-valued form to define the logical parallax in human thinking, and which are useful in their many-valued forms to interpret the lower forms of intelligence in animals, plants, et cetera.

But it should never be forgotten that, no matter how low or how high a certain intelligence level is, its essential structure is two-valued Aristotelian. This goes for the systems of T-plurality also. For an intelligence with the range F-T" the reduction-values "T" and "T'" are no genuine values. They only indicate procedures by which the wider range of F-T" can be reduced to the requirements of the lower intelligence forms F-T and F-T'.

Here a final question is in order: Let it be assumed that there are no higher forms of individual intelligence in the world than that of the human race. Is there any other possibility of applying logical calculi of T-plurality? Indeed, there seems to be at least one. The general structure of the universe can probably be described as a T-plurality-system. Everybody knows the biblical formula: "In the beginning God created the heaven and the earth ..." For the logician this statement is a mythological version of the semantically equivalent proposition: the universe reflects for the human observer an intelligent pattern of a logical order that needs for its proper definition a system of T-plurality.

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